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A HYDRODYNAMICALLY ACTIVATED ROTATIONAL BALANCING SYSTEM

A THESIS

Presented to

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Joseph Gordon Tigner

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A HYDRODYNAMICALLY ACTIVATED ROTATIONAL

BALANCING SYSTEM

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SUMMARY

Unbalance in rotating machinery can cause the transmission of large forces to the supports of the machine. In a washing machine such forces lead to excessive vibration and can cause the machine to "walk" across the floor. The unbalance, created by a varying amount of clothes unevenly distributed around the tub, becomes a problem during the high speed spin cycle when water is extracted from the clothes. At maximum spin speed an inertia force as high as 700 lbf. can be generated. The current production model of the Whirlpool horizontal axis combination washer/dryer achieves balance by injecting water into one or more of three tanks, equally spaced around the circumference of the tub. This system is adequate but is more costly than desired, is not sensitive to changes in unbalance once water has been injected into the tubs, and generally takes two to three minutes to achieve adequate balance.

The object of this research was to investigate the feasibility of balancing this inertia force by moving the tub relative to the center of the rotating shaft. The tub is moved by hydraulic cylinders activated by pressure changes in a journal bearing supporting a portion of the load transmitted through the shaft. Translatory motion of the tub occurs until the inertia force, and hence the rotating pressure profile, disappears. The proposed model predicts balance in just over 30 seconds while restricting the maximum inertia force to less than 80 lbf.

The research was entirely analytical in nature and was primarily concerned with an optimization of the many design parameters involved in the analysis. Finally, recommendations are made concerning the construction of a physical model of the proposed system.

NOMENCLATURE

a	distance from ball bearing to journal bearing in design one, in.
b	distance between ball bearings in design one, in.
c	radial clearance between shaft and ball bearing, in.
d	diameter of pipe in fluid flow, in.
d_1	diameter of tube in line one, in.
d_2	diameter of tube in line two, in.
d_c	diameter of capillary tube, in.
e	eccentricity; offset of shaft in journal bearing, in.
g	acceleration due to force of gravity, in/sec^2.
h	thickness of fluid film in journal bearing, in.
h_t	time interval in numerical solution, sec.
m	mass of unbalanced clothes, lbf-sec^2/in.
p	fluid pressure, psi.
p_{max}	maximum pressure developed in journal bearing, psi.
r	radius of shaft, in.
r_i	inner radius of elastic tube, in.
r_b	radius of elastic tube at any time t, in.
r_c	radius of the capillary tube, in.
t	thickness of elastic tube, in; and time, sec.
u	displacement of elastic tube, in.
x	direction of tangential fluid flow in journal bearing.
y	direction of radial flow in journal bearing.

z	distance of unbalanced clothes from rear ball bearing, in.
A_1	cross-sectional area of tubing in lines one and two, in^2.
A_s	surface area of pistons, in^2.
B_s	distance from ball bearing to journal bearing in design two, in.
D	inside diameter of journal bearing, in.
E	elastic modulus, psi.
F	force on shaft due to unbalanced clothes, lbf.
F_1	portion of total inertia force due to unbalanced clothes, lbf.
F_2	portion of total inertia force due to offset of tub center, lbf.
FX_1	portion of total inertia force acting in X_1 direction, lbf.
FX_2	portion of total inertia force acting in X_2 direction, lbf.
I	second moment of inertia of shaft cross-section, in^4.
K_{ij}	$i,j = 1$-4, parameters in Runge-Kutta solution of differential equations.
L	length of journal bearing, in.
L_1	length of tubing in line one, in.
L_2	length of tubing in line two, in.
L_b	length of elastic tube, in.
L_c	length of capillary tube, in.
M	mass of tub, clothes, and water, lbf-sec^2/in.
P_1	pressure at hole 1 in shaft, psi.
P_2	pressure at hole 2 in shaft, psi.
P_3	pressure at hole 3 in shaft, psi.
P_4	pressure at hole 4 in shaft, psi.
P_1^*	pressure on piston surface in cylinder chamber 1, psi.

P_2^* pressure on piston surface in cylinder chamber 2, psi.

P_3^* pressure on piston surface in cylinder chamber 3, psi.

P_4^* pressure on piston surface in cylinder chamber 4, psi.

P_B pressure at elastic tube upstream from cylinder, psi.

P_G pressure at elastic tube downstream from cylinder, psi.

P_L load per projected bearing area, psi.

Q flow of oil between shaft and journal bearing, in^3/sec.

Q_s flow lost to side leakage in journal bearing, in^3/sec.

R_1 reaction of journal bearing on shaft in design two, lbf.

R_2 reaction of ball bearing on shaft in design two, lbf.

R_a reaction of rear ball bearing on shaft in design one, lbf.

R_b reaction of front ball bearing on shaft in design one, lbf.

Re Reynold's number = $\frac{\gamma V d}{\mu g}$.

R_f radius of unbalanced clothes from tub center, in.

R_o reaction of journal bearing on shaft in design one, lbf.

S Sommerfeld number = $\frac{\mu\omega}{P_L}\left(\frac{r}{c}\right)^2$

V relative velocity of moving surface in a journal bearing, in/sec.

X_1 motion of the piston in cylinder one, in.

X_2 motion of the piston in cylinder two, in.

Y_1 velocity of the piston in cylinder one, in/sec.

Y_2 velocity of the piston in cylinder two, in/sec.

β location of tub center from cylinder one, degrees.

γ weight density of fluid, lbf/in^3.

δ displacement of tub center from shaft center, in.

δ_1 deflection of shaft due to the inertia force alone, in.

δ_2 deflection of shaft due to bearing reaction alone, in.

ϵ $\frac{e}{c}$ = eccentricity ratio.

ϵ_θ circumferential strain in elastic tube, in/in.

θ location of unbalanced clothes, degrees.

μ viscosity of fluid, reyns = $lbf\text{-}sec/in^2$.

ν Poisson's ratio for elastic tube.

σ_θ circumferential stress, psi.

σ_r radial stress, psi.

τ loss parameter, sec.

ϕ angle at which pressure in journal bearing is desired, degrees.

ψ attitude angle, degrees.

ω angular speed of shaft, radians/sec.

CHAPTER I

INTRODUCTION

Background

The problem of balancing rotating machinery has been with industry from its beginning. Particular difficulty has been encountered in devising a balancing system that works efficiently when both the magnitude and the direction of the unbalance are not constant. Such is the case with a washing machine where the unbalance becomes a problem during the high-speed spin cycle when water is removed from the clothes. The large inertia force created by the uneven distribution of clothes leads to excessive cabinet motion, which can cause the machine to "walk" across the floor. The object of this thesis is to provide evidence for the justification of further development of a proposed system to mechanically balance this inertia force.

History of the Problem

Various types of balancing schemes, both mechanical and electromechanical, have been investigated at the Whirlpool Advanced Engineering Laboratories in St. Joseph, Michigan. According to Twitchell [1] there are three basic means of mechanically balancing a tub: (1) movement of mass within the rotating unit, (2) addition or subtraction of mass at various locations in the rotating unit, (3) relocation of the center of rotation to coincide with the center of mass. The current model of the Whirlpool horizontal axis combination washer/dryer employs the second of these methods for balancing. Clothes are washed and tumbled at low speed and have some of the water in the clothes removed by centrifugal force during high-speed spin after which the remaining water is removed by hot air drying at low tumble speed. To balance the tub during the spin cycle, a mechanical system activated by cabinet motion injects water into one or more of three tanks equally spaced around the circumference of the tub. This system is more costly than desired and is not sensitive to changes in the inertia force once water is injected into the tanks.

A literature search of material available from the Whirlpool Corporation library revealed many balancing schemes which generally have had limited degrees of success [1,2,3]. Most of these proposals have been concerned with balancing either the horizontal axis or vertical axis machines. Thus there is the need for a balancing system which is applicable to both types of washing machines.

A common problem with previous attempts to balance the tub is the method of sensing the unbalance. Most of these designs utilize unwanted machine motion to activate the balancing system. Twitchell concluded that such radial displacement of the tub assembly cannot be effectively used as the control parameter for activating a sensing device over a wide range of speeds and loads because of an inherent phase lag between the direction of the unbalance and the actual direction of the tub excursion. The one parameter, he notes, which bears an exact relationship to the magnitude and direction of the unbalanced load is the centrifugal force created by the unbalance. This research was centered around using changes in the system brought about directly by this parameter to activate a mechanical system that brings about a complete balance of the system. If the unbalanced mass of clothes lies longitudinally in the center of the tub, there is a complete static and dynamic balance of the system. The maximum unbalanced moment is around 50 in-lbf, and occurs when the unbalanced clothes lie in the extreme front or rear of the tub.

The Proposed System

The most prominent changes in the system brought about by an unbalanced load are vibratory motion of the machine frame, deflection of the shaft-tub arrangement, and a change in load carried by the support bearings. It was the object of this research to utilize the change in load carried by the support bearings as an indication of the amount and direction of the unbalance.

The proposed system achieves balance by relocation of the center of mass to coincide with the center of rotation. The tub is moved with respect to the shaft by hydraulic cylinders which are activated by pressure changes in a journal bearing supporting a load transmitted to it through the shaft. The load-supporting pressure profile is created by the flow of lubricant between the shaft and the journal bearing. If the load carried is due to the inertia force of the unbalanced clothes, then it is a rotating load relative to the bearing and is distinguishable from unidirectional loads such as gravity. The pressure profile created by such a rotating load is also rotating with respect to the bearing and is a function of both the magnitude and direction of the inertia force. Pressure differences at fixed locations around the shaft activate the balancing mechanism, and translatory motion of the tub relative to the shaft occurs until the inertia force disappears. The literature review revealed no attempts to take direct advantage of the forces transmitted to the support bearings to indicate the amount and location of unbalance.

Method of Attack

The research was entirely analytical in nature and was based on a mathematical model of the proposed control system, involving principles of machine dynamics, fluid mechanics, lubrication, deflection analysis, and numerical mathematical analysis. The equations of motion of the tub were derived and solved numerically on the UNIVAC 1108 digital computer. The computer program is included as a design tool for any future work. Emphasis was placed on variation of the many parameters involved in the control system to determine an optimum range of the parameters that would allow the balancing system to perform effectively.

CHAPTER II

THE PROPOSED DESIGN

General Concept

A properly designed balancing system must be sensitive to unbalance equally in all directions over a wide range of loads and speeds. The mechanical balancing system proposed in this thesis is a closed-loop control network which is described only partially by empirical relationships. Although equations are derived to describe the flow of fluid in the hydraulic lines and the motion of the tub, exact solutions for the flow of fluid in a journal bearing do not exist, and we must rely on previously obtained numerical solutions in order to describe such flow.

Figure 1(a) shows a tub at rest with a mass of clothes offset from the tub center, while Figure 1(b) shows the tub in a balanced configuration. The center of the tub is offset an amount needed to generate a centrifugal force equal and opposite to the centrifugal force caused by the unbalanced clothes. The movement of the tub is accomplished by hydraulic cylinders connected to holes in the shaft. The existence of an inertia force creates a rotating pressure profile which establishes a pressure difference on the sides of the pistons in the cylinders, thereby inducing motion of the pistons. This motion continues until the inertia force disappears at which time the tub is rotating eccentrically around the shaft completely balanced.

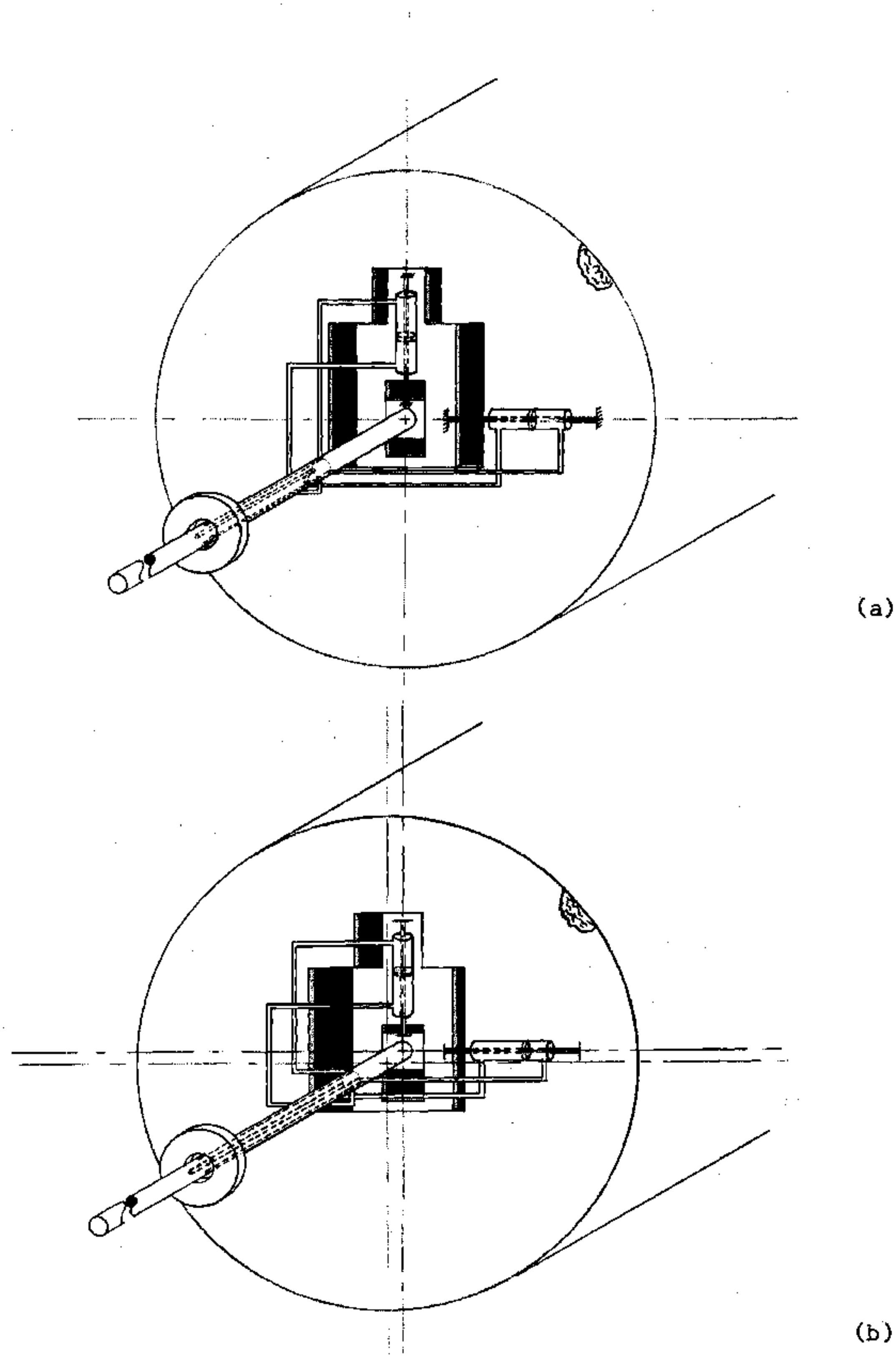

Figure 1. Proposed Balancing System Before and After Balance

The entire tub-shaft arrangement consists of three rigid bodies shown in Figure 2. The hydraulic cylinders are aligned 90 degrees apart, and tub motion occurs independently in the directions of piston motion in each of these cylinders. Torque is transmitted from the shaft to the tub by dovetail joints which restrain the motion of the tub to a plane perpendicular to the shaft (see Figure 3).

Equations of Motion of the Tub

To describe the motion of the tub, we need the forces acting on the tub. Examining a simplified free body diagram of the forces on the tub in the X_1 direction (see Figure 4), we see that there are two contributions to the forces acting on the tub. These forces are due to a pressure difference $(P_1^*-P_3^*)$ acting on surface area A_s and the portion of the total inertia force acting in the X_1 direction. There are other forces acting on the tub; namely, friction forces due to motion between the movable parts of the system, a force from Coriolis acceleration of the tub, and a term accounting for the angular acceleration of the tub. Based on solutions obtained neglecting these forces, it was determined that these forces do not appreciably affect the motion of the tub. The equation of motion of the tub in the X_1 direction is thus

$$M \frac{d^2X_1}{dt^2} = (P_1^*-P_3^*)A_s - FX_1 \tag{1}$$

where FX_1 is the portion of the total inertia force acting in the X_1 direction.

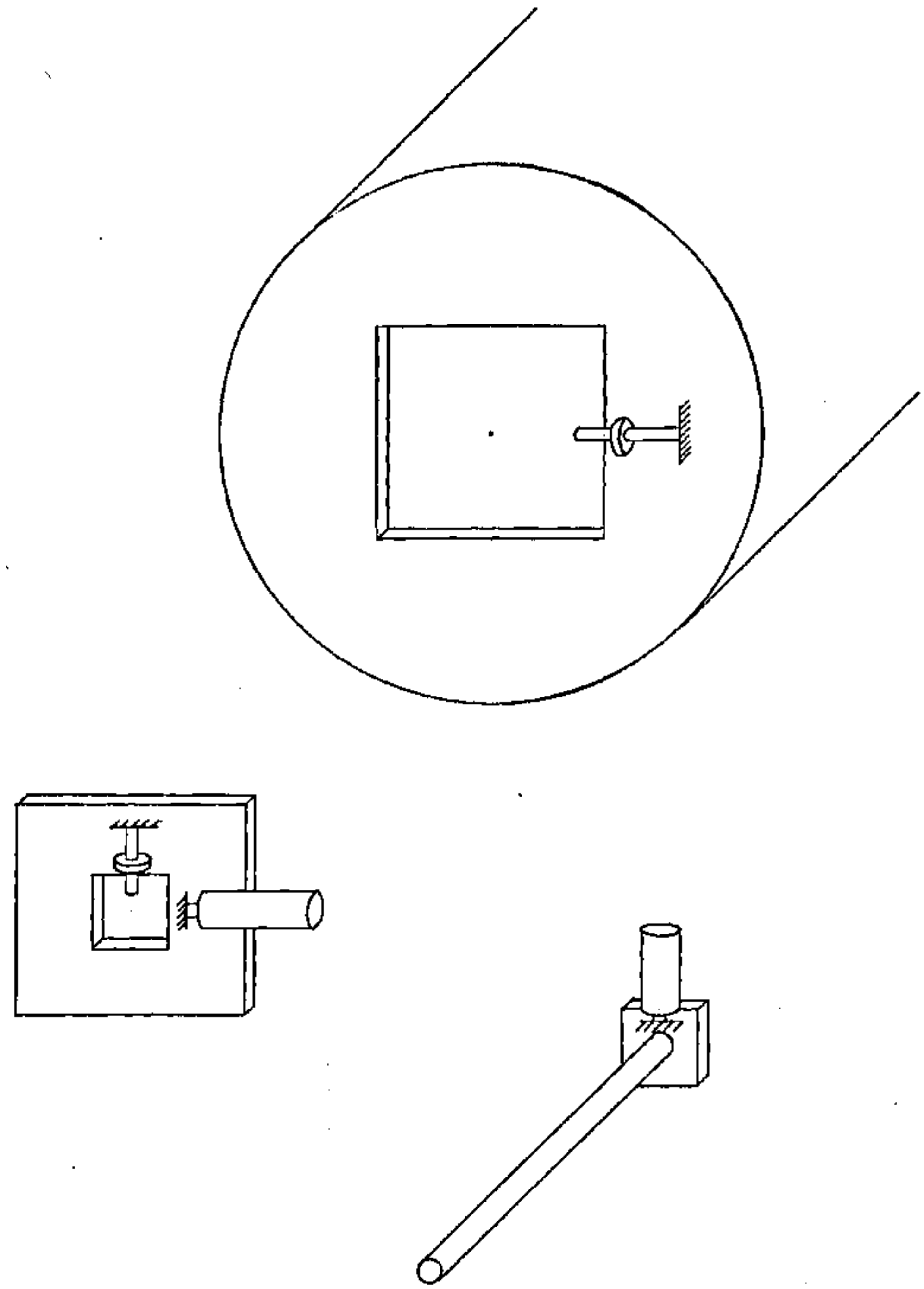

Figure 2. Three Rigid Bodies in Balancing System

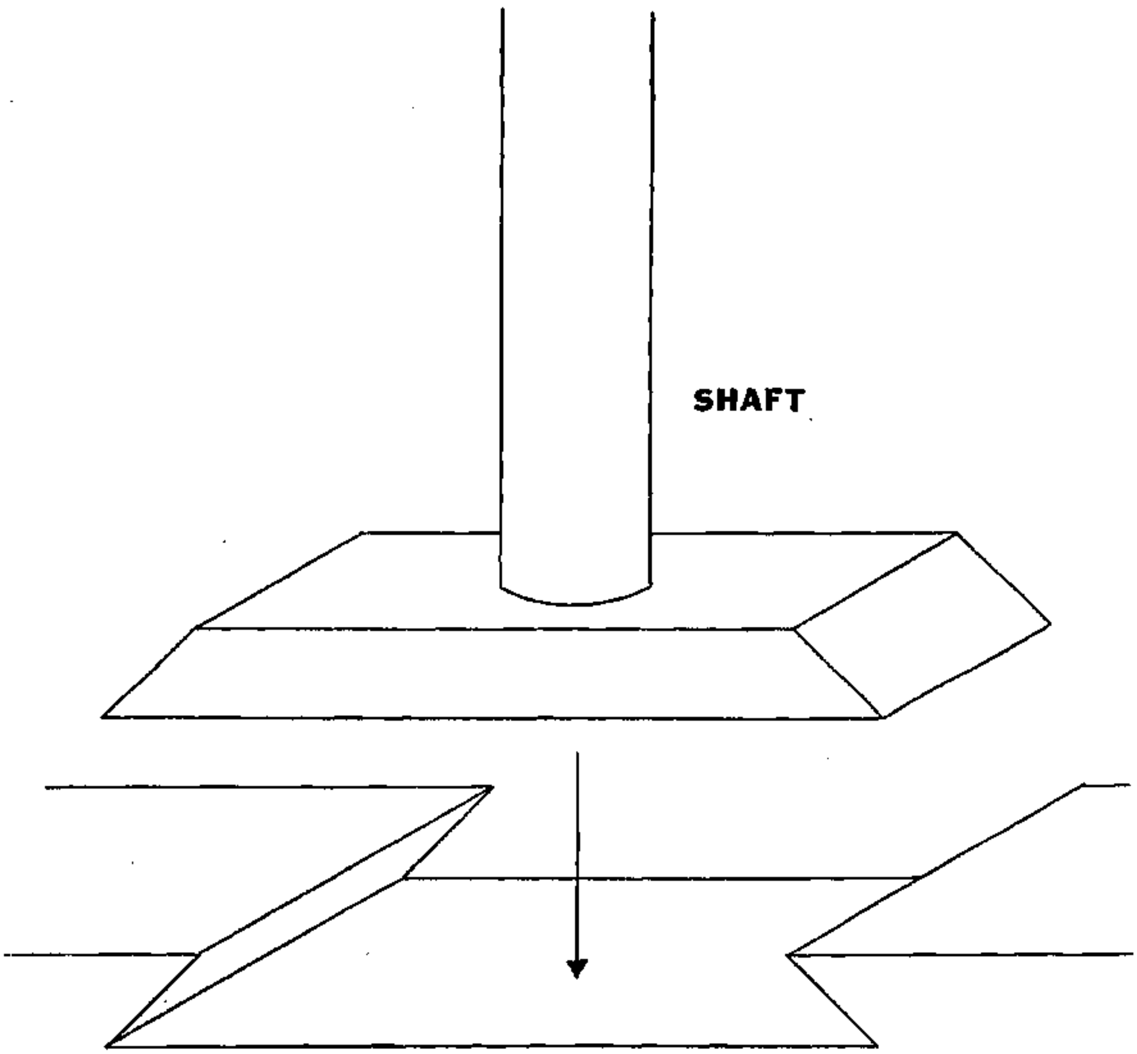

Figure 3. Connection of Shaft to Tub to Allow
Transmission of Shaft Torque

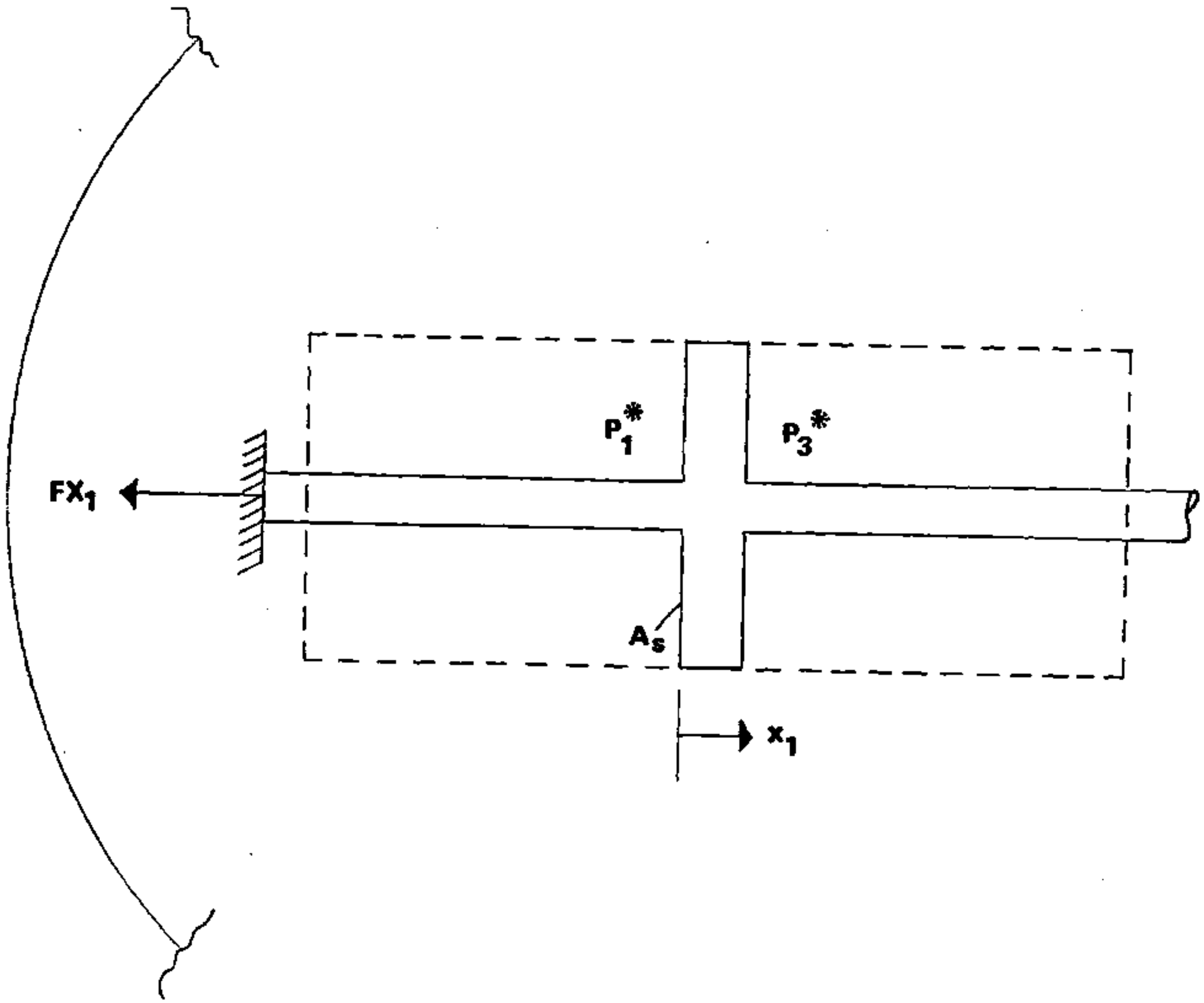

Figure 4. Free Body of Forces in X_1 Direction

Similarly, for motion in the X_2 direction

$$M \frac{d^2X_2}{dt^2} = (P_2^* - P_4^*)A_s - FX_2 \tag{2}$$

The motion of the tub center relative to the rotating shaft is

$$\delta = (X_1^2 + X_2^2)^{\frac{1}{2}} \tag{3}$$

from the center of the shaft at an angle

$$\beta = \tan^{-1}\left(\frac{X_2}{X_1}\right) \tag{4}$$

The terms on the right side of equations (1) and (2) are all functions of time. We can derive expressions for the inertia force components explicitly as a function of time, while the pressure terms are obtained by analysis of the equations of flow into and out of the cylinder.

Determination of the Inertia Force

The total inertial force is the vector sum of two inertial forces--one due to clothes offset from the center of the tub and one due to the displacement of the tub center from the center of the shaft. The components of the total inertial force acting in directions X_1 and X_2 are the vector sums of the two individual inertial forces in the X_1 and X_2 directions, respectively.

Figure 5 shows the tub in an unbalanced configuration. The coordinate system is fixed to the rotating shaft.

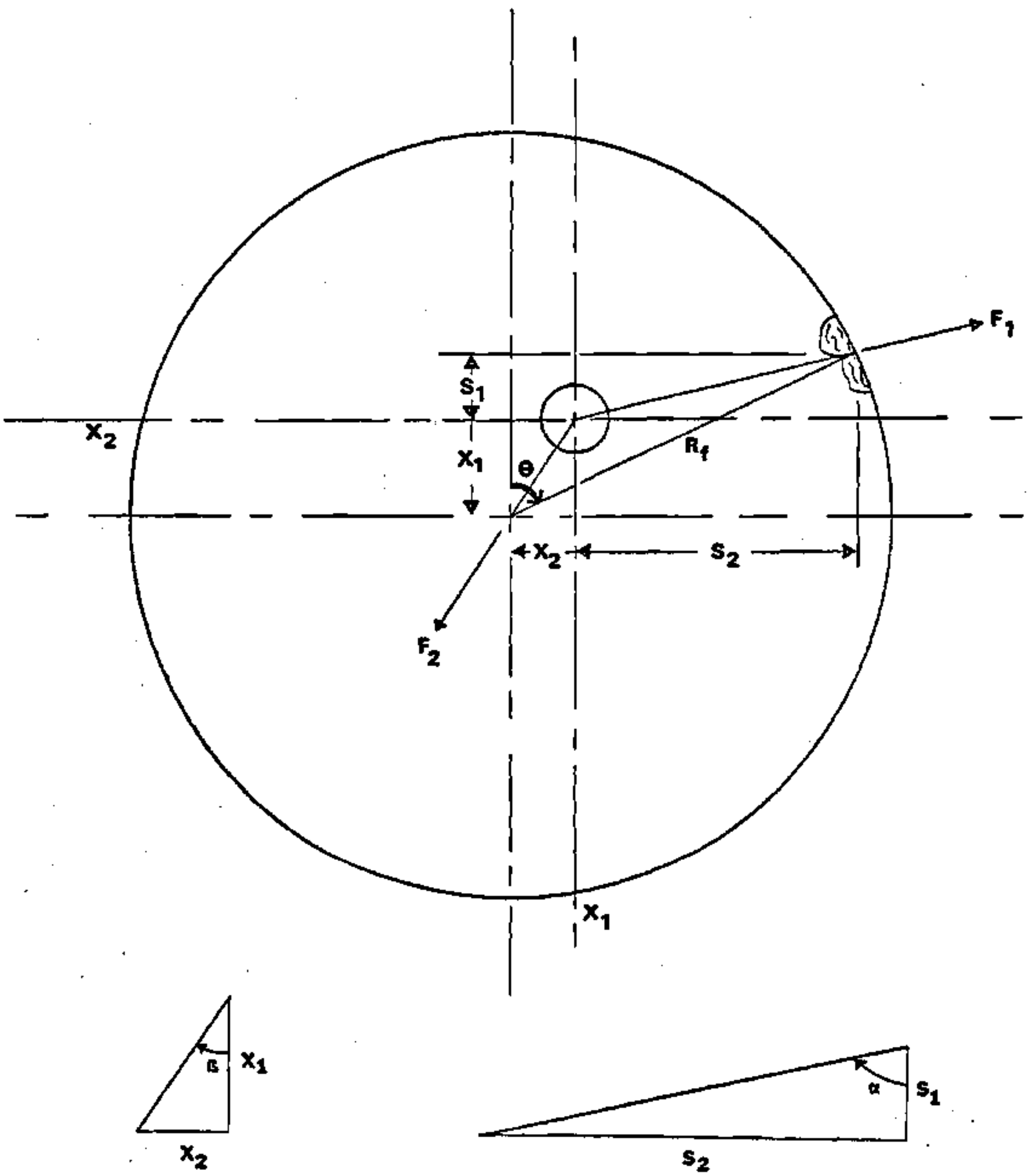

Figure 5. Tub in an Unbalanced Configuration

We define

R_f = radius of the unbalanced clothes from the tub center, in.

θ = location of unbalanced clothes clockwise from cylinder 1, degrees.

$S_2 = R\sin\theta - X_2$.

$S_1 = R\cos\theta - X_1$.

Thus the magnitudes of the two inertial forces are

$$F_1 = m\omega^2(S_1^2+S_2^2)^{\frac{1}{2}}$$

$$F_2 = M\omega^2(X_1^2+X_2^2)^{\frac{1}{2}}$$

where

F_1 = inertia force due to unbalanced clothes, lbf.

F_2 = inertia force due to displacement of tub center, lbf.

Therefore

$$FX_1 = F_1\cos\alpha - F_2\cos\beta$$

$$FX_2 = F_1\sin\alpha - F_2\sin\beta$$

where

$\alpha = \arctan(S_2/S_1)$

$$\sin\alpha = \frac{S_2}{(S_1^2+S_2^2)^{\frac{1}{2}}}$$

$$\cos\alpha = \frac{S_1}{(S_1^2+S_2^2)^{\frac{1}{2}}}$$

$$\sin\beta = \frac{X_2}{(X_1^2+X_2^2)^{\frac{1}{2}}}$$

$$\cos\beta = \frac{X_1}{(X_1^2+X_2^2)^{\frac{1}{2}}}$$

and thus

$$FX_1 = m\omega^2 S_1 - M\omega^2 X_1 \tag{5}$$

$$FX_2 = m\omega^2 S_2 - M\omega^2 X_2 \tag{6}$$

These expressions may now be substituted into equations (3) and (4), respectively.

Forces Transmitted to the Bearings

In order to determine the pressures generated in the journal bearing we must first know what portion of the load due to the inertia force is carried by the journal bearing. This is dependent on the location of the journal bearing relative to the other supports. Two designs have been considered in this research--one in which the journal bearing is placed between two self-aligning ball bearings and one in which the journal bearing and only one ball bearing carry the load.

The first of these is an attempt to adapt the balancing mechanism to the current machine. The free body diagram in Figure 6 reveals the

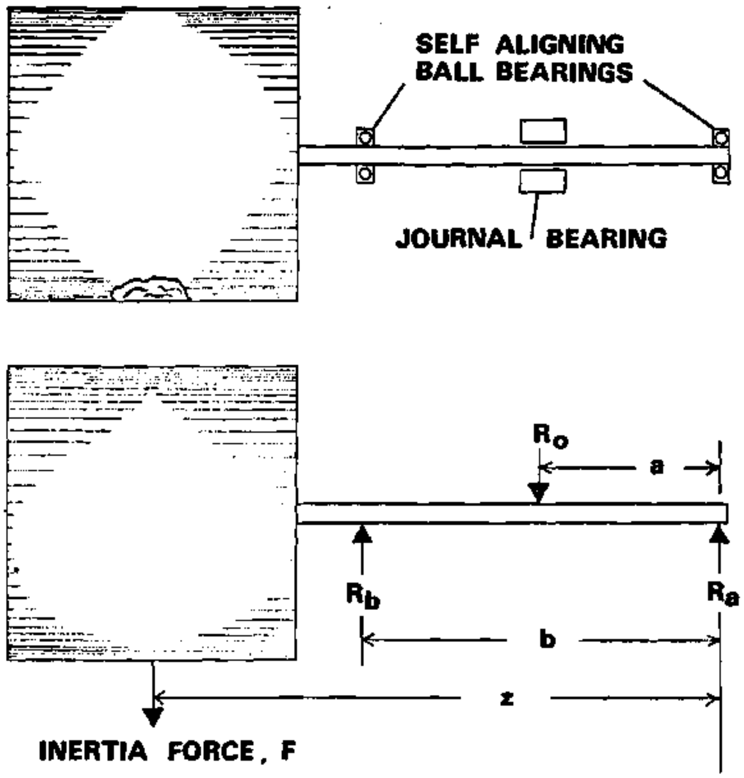

Figure 6. Free Body Diagram of Forces on Shaft in Three-Support Design

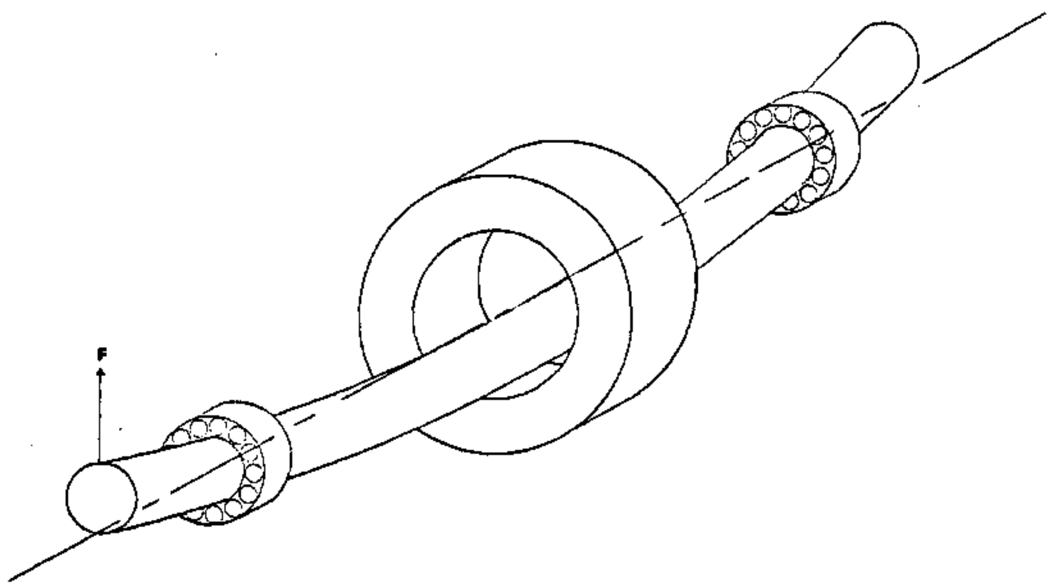

Figure 7. Position of Shaft Relative to Journal Bearing in Three-Support Design

shaft to be a statically indeterminant beam, simply because it has more supports than are necessary to maintain equilibrium. Standard methods of obtaining the reactions for statically indeterminant beams cannot be used to obtain an exact solution in this analysis, for though we can assume zero deflection of the shaft at the two ball bearings, a similar assumption at the journal bearing is not as legitimate. The shaft does not rotate concentrically inside the bearing, but is offset by an amount e, the eccentricity, which is determined from lubrication analysis. Further complicating the problem is the fact that the line of centers of the shaft and journal bearing is not in the same plane as the applied load (see Figure 13). An approximation can, however, be made to the load on the journal bearing by assuming zero deflection at the journal bearing. Figure 7 shows the position of the shaft relative to the bearing.

Let the journal bearing reaction R_o in Figure 6 be the redundant reaction. Using the principle of superposition of forces, we determine the deflection δ_1 at the journal bearing due to the inertia force F alone, then determine the force R_o needed to bring the deflection of the shaft δ_1 back to zero. The true load carried by the journal bearing lies somewhere between $0 < R_{true} < R_o$.

The coordinate system is fixed to the rotating shaft, and deflection is positive upward in Figure 8. We neglect the weight of the shaft. Deflection due to F alone is

$$\delta_1 = \frac{F(z-b)ab}{6EI}\left(1 - \frac{a^2}{b^2}\right) \qquad (7)$$

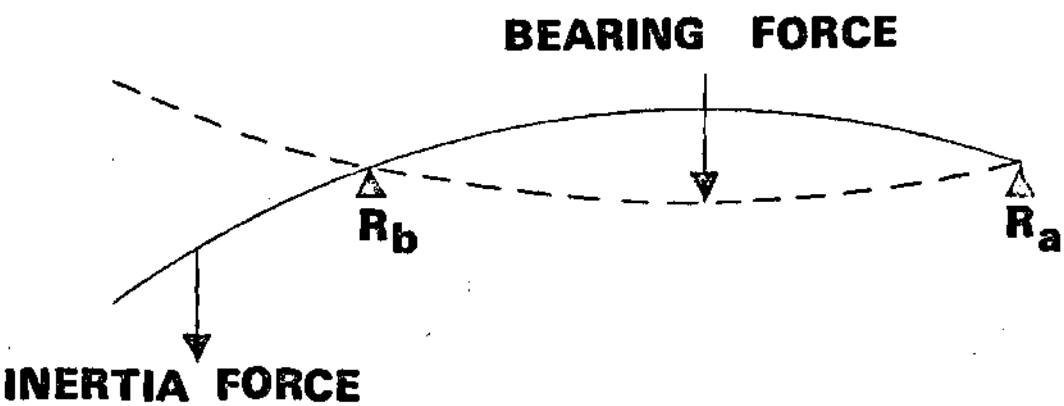

Figure 8. Deflection of Shaft Due to Forces on Shaft in Three Support Design

Due to the journal bearing reaction force R_o, the deflection is

$$\delta_2 = \frac{-R_o(b-a)a(2ab-2a^2)}{6EIb} \tag{8}$$

The algebraic sum of these deflections is the total deflection which is assumed to be zero at the journal bearing. So

$$0 = \delta_1 + \delta_2 = \frac{F(z-b)ab\left(1 - \dfrac{a^2}{b^2}\right)}{6EI} - \frac{R_o(b-a)a(2ab-2a^2)}{6EIb} \tag{9}$$

The only unknown is R_o and

$$R_o = \frac{F(z-b)ab^2\left(1 - \frac{a^2}{b^2}\right) - 6\delta_2 EIb}{a(b-a)(2ab-2a^2)} \tag{10}$$

The use of R_o in the lubrication analysis will cause predictions of pressure larger than do actually exist because $R_o > R_{true}$. For this reason the two-support design, in which the exact load carried by the bearing can be determined, is used in this analysis.

Figure 9 shows the shaft supported by a ball bearing and a journal bearing a distance B_s from the ball bearing. The relative position of the shaft and bearing is shown in Figure 10. Summing moments about the ball bearing, the reaction R_a is

$$R_a = F \frac{z}{B_s} \tag{11}$$

The sensitivity of the balancing system is influenced by the ratio z/B_s; the larger this ratio, the greater the bearing reaction for a given inertia force, and the larger the pressures generated to activate the balancing mechanism.

Thus the advantage of using the two-support design in the analysis is the accuracy in analytically determining the load. However, the journal bearing for this design is susceptible to much greater wear than in the three-support design, primarily due to the weight of the tub. In the three-support design the journal bearing can be placed

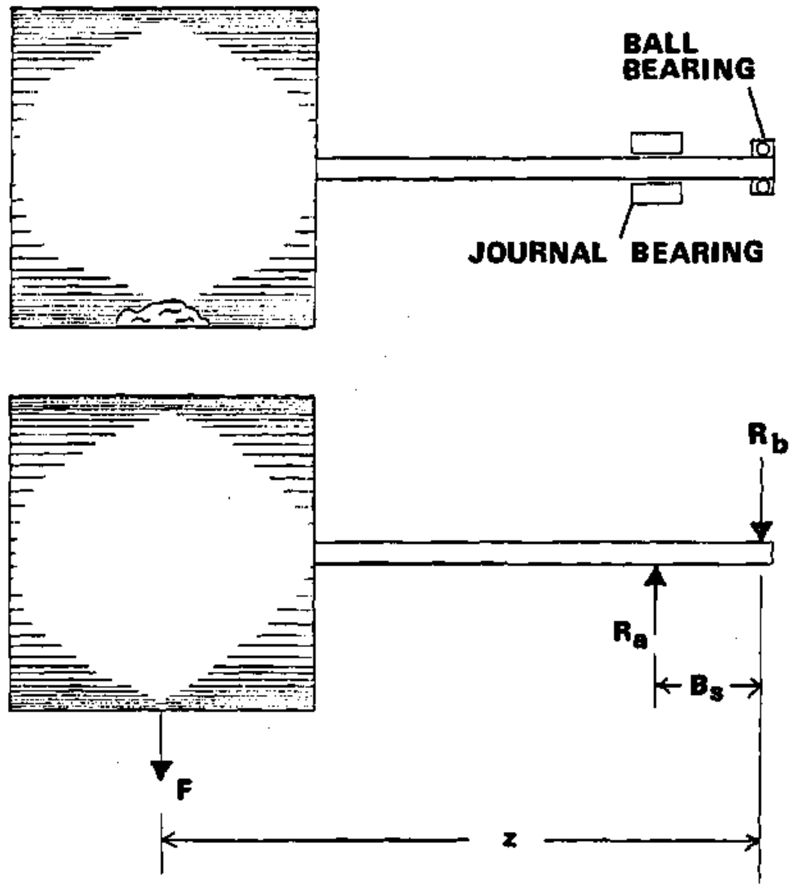

Figure 9. Free Body Diagram of Forces on Shaft in Two-Support Design

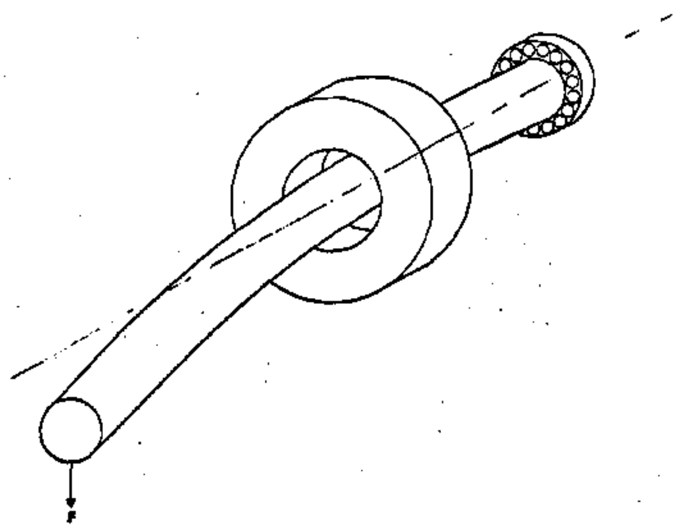

Figure 10. Position of Shaft Relative to Journal Bearing in Two-Support Design

concentrically with the shaft with the machine at rest, to render the bearing insensitive to the force of gravity. This cannot, however, be done in the two-support design.

Journal Bearing Analysis

A journal bearing supports a load transmitted through a shaft by a fluid film created by the flow of lubricant between two surfaces. The existence of this film is dependent upon the combination of speed, load, and lubricant viscosity being within a broad but definite range. Insufficient surface area, increased load, lubricant breakdown, surface velocity decrease all can lead to a film thickness insufficient to prevent contact between the surfaces. When this occurs, the bearing is operating under boundary, or thin film lubrication and wear becomes of greater concern.

The number of parameters in bearing design is large. Among these are the lubricant used, the load carried by the bearing, the machine operating speed, and the bearing dimensions. Other factors which are dependent on the above list include the friction coefficient, the temperature rise, the ability of the lubricant to carry away heat, the oil flow, and the minimum film thickness.

The designer is aided by the definition of some dimensionless variables which relate the above factors:

(a) ϵ = eccentricity ratio = $\frac{e}{c}$

(b) S = Sommerfeld No. = $\frac{\mu\omega}{P_L}\left(\frac{r}{c}\right)^2$

(c) $\frac{Q}{rc\omega L}$ = flow variable

(d) Q_s/Q = side flow variable determining the ratio of side leakage to total flow

(e) p_{max}/P_L = ratio of maximum pressure developed in the bearing to the load per projected bearing area

With these definitions we can study the effect of independently varying each variable and thus design the optimum bearing for a given set of operating conditions.

The solution to Reynold's equation describing the flow in a journal bearing has been obtained numerically for bearing length/diameter ratios of 1/4, 1/2, 1 and a few others [4,5,6]. For this investigation, the ratio L/D = 1 was chosen and the solutions hereafter apply only to a journal bearing of such relative dimensions. Table 1 gives values of the above design variables for various eccentricity ratios [5,6].

Table 1. Relationship of Design Variables in a Journal Bearing with L/D = 1

ϵ	S	$\frac{Q}{rc\omega \ell}$	$\frac{Q_s}{Q}$	$\frac{p_{max}}{P_L}$	ψ
0	∞	3.2	0	1.0	86
.1	1.35	3.35	.15	1.85	79
.2	.632	3.55	.28	1.89	74
.3	.382	3.75	.4	1.93	68
.4	.261	4.0	.49	2.08	62
.5	.179	4.15	.59	2.23	56
.6	.120	4.3	.68	2.41	50
.7	.0765	4.45	.77	2.69	43
.8	.0448	4.6	.84	3.15	36
.9	.0191	4.7	.92	4.10	25
1.0	0	4.85	1.0	10.0	0

Cameron [5] presents a graph relating ϵ and S for low eccentricity ratios ($0<\epsilon<.1$), which is generally the range of values found in this investigation. The normal range of ϵ in journal bearing operation is $.2 < \epsilon < .4$, and the data available relating the above defined parameters is in the range $.1 < \epsilon < 1$. Extrapolation is therefore necessary to obtain values for the dimensionless parameters earlier defined in the range of ϵ found in this analysis.

Reynold's equation describing flow through a journal bearing assuming an incompressible lubricant is [6]

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial y}\right) = -6V\,\frac{\partial h}{\partial x} \qquad (17)$$

Also assuming the lubricant obeys Newton's law of viscous flow, fluid inertia forces are negligible, and the viscosity of the fluid is constant throughout the film, we can obtain the pressures around the shaft by interpolation of previously obtained numerical solutions of equation (17). The flow of oil is based on oil supplied to the bearing at atmospheric pressure and on the absence of holes or grooves in the bearing. The boundary conditions upon which the solutions are based are the Reynold's boundary conditions in lubrication theory [6], which assume that zero gage pressure exists in the diverging region where the Sommerfeld boundary conditions predict negative pressures, simply because the fluid cannot support a tensile load. The basic form of the pressure profile is shown in Figure 11, the exact shape depending upon the bearing operating conditions.

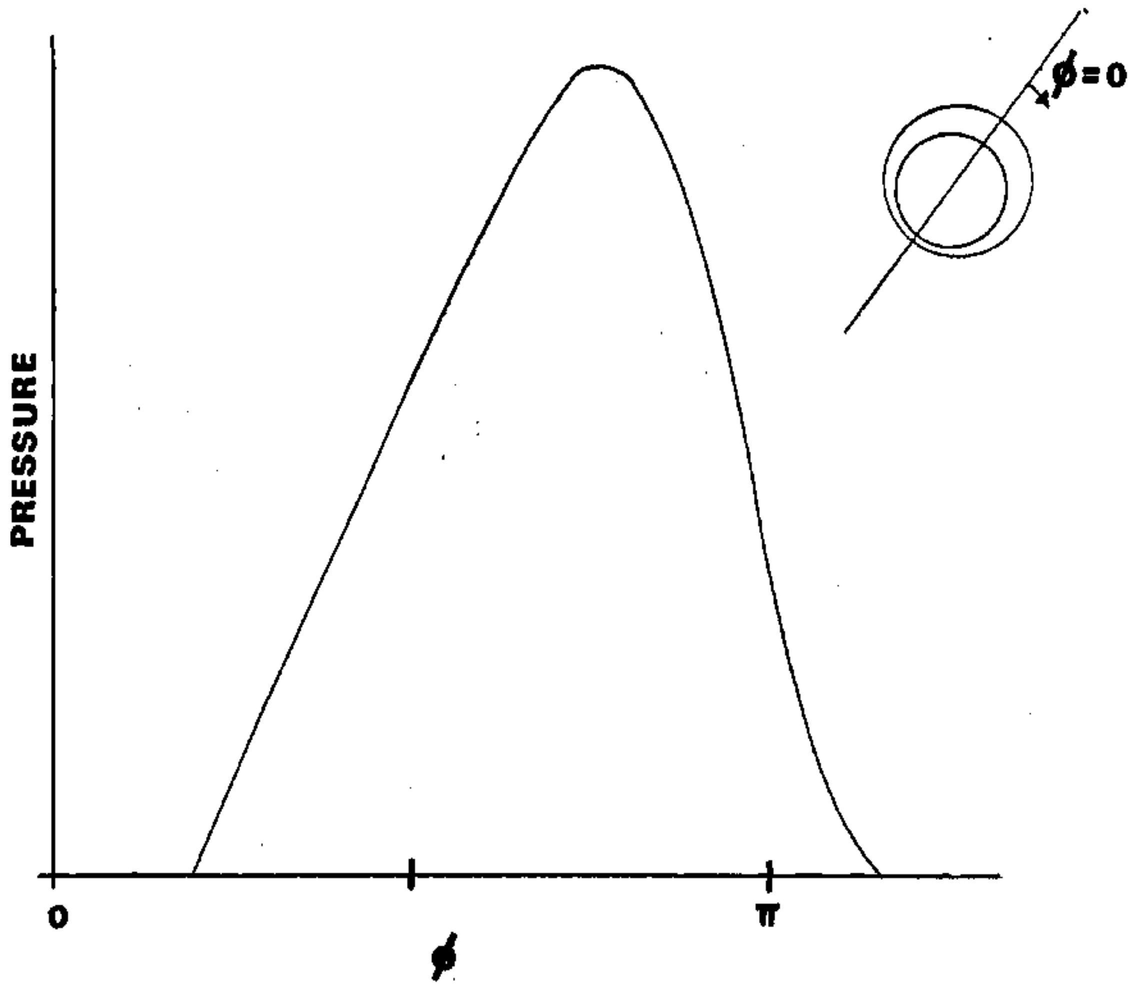

Figure 11. Pressure Profile Generated in a Journal Bearing

The hydraulic cylinders which move the tub are connected to the shaft by hydraulic line as shown in Figure 12. The holes in the shaft are directly aligned with the cylinders to which they are connected as shown in Figure 13. In order to determine the pressures exerted on the pistons, we must first determine the pressures at each of the holes in

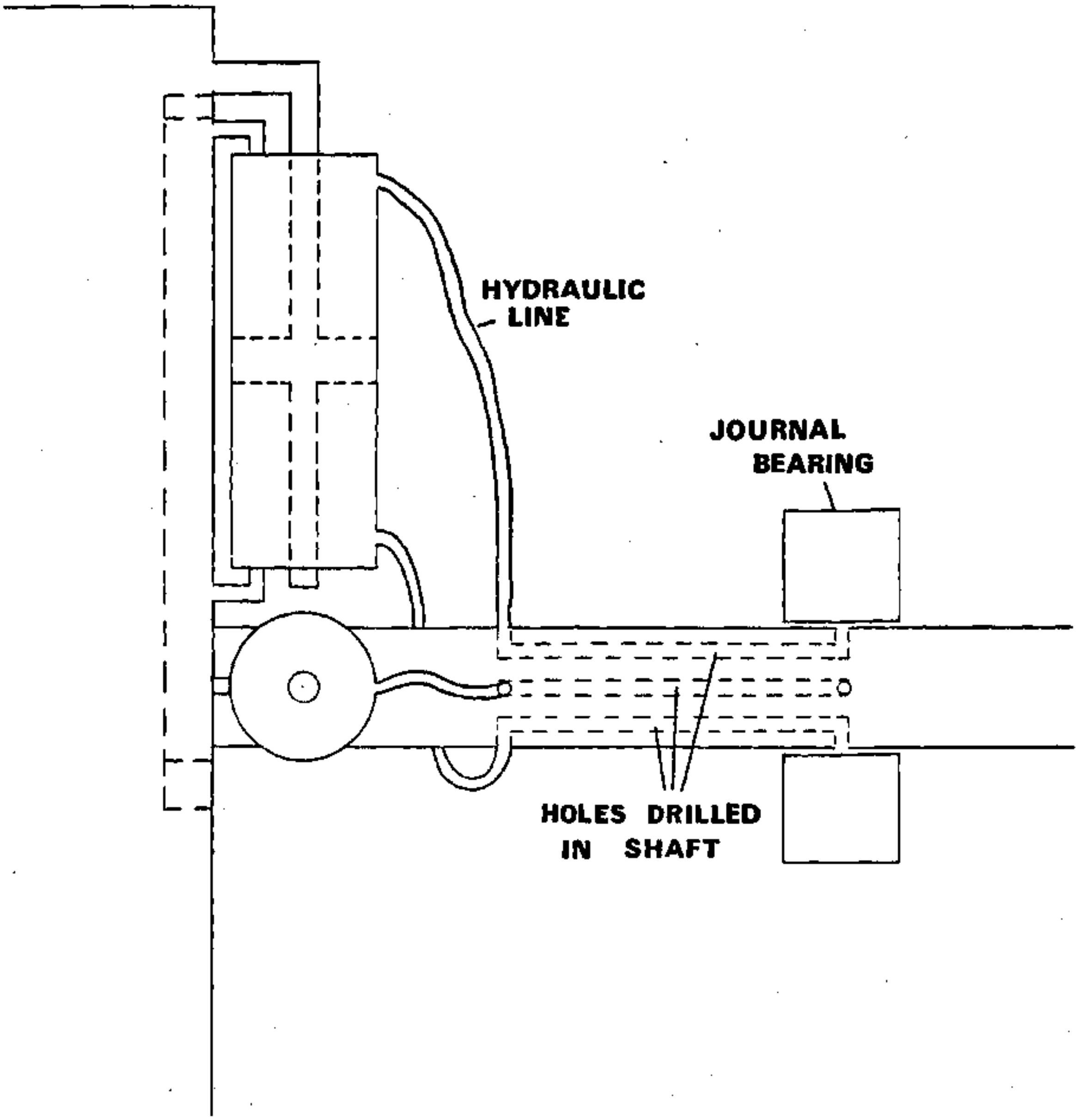

Figure 12. Side View of Proposed Balancing System

(a)

(b)

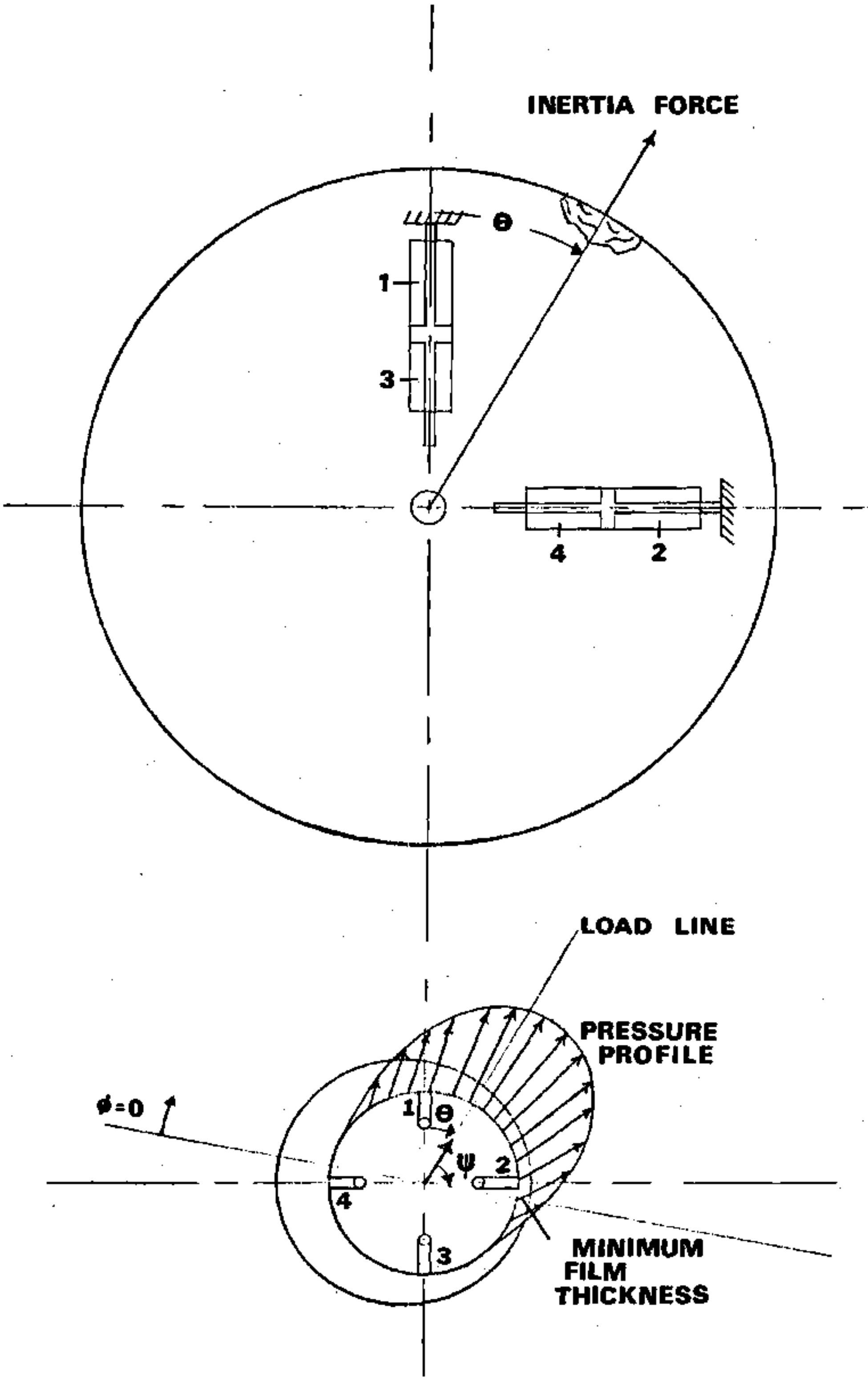

Figure 13. Location of Shaft Relative to Journal Bearing for Unbalance at θ Degrees

the shaft. For a given inertia force and machine speed the pressures at each of the holes is constant because the pressure profile due to the rotating inertia force is itself rotating.

Figure 13(a) shows an unbalanced load of clothes θ degrees in the direction of tub rotation from cylinder 1. The position of the shaft relative to the journal bearing is shown for such a load. This is, of course, an instantaneous position since the shaft center is rotating about the center of the bearing.

In accordance with journal bearing theory we measure the location ϕ at which we desire the pressure from the line of centers of the tub and shaft, the location of $\phi = 0$ being 180° from the point of minimum film thickness. The attitude angle ψ is defined as the oblique angle between the line of centers and the line of action of the applied load. Referring to Figure 13(b) we see that the locations of the holes from $\phi = 0$ are

$$\begin{aligned} \phi_1 &= 180 - (\psi+\theta) \\ \phi_2 &= 270 - (\psi+\theta) \\ \phi_3 &= 360 - (\psi+\theta) \\ \phi_4 &= 90 - (\psi+\theta) \end{aligned} \tag{18}$$

where ϕ_i = location of hole i, degrees, i = 1 - 4. Given the machine operating conditions and bearing dimensions, we can now determine the pressure developed at any location around the shaft.

Flow Through Hydraulic Lines

In order to write the equations of motion of the pistons, expressions for the pressures on both sides of the pistons must be obtained. The changes in pressure sensed at the journal bearing are transferred to the cylinders by the hydraulic fluid between the journal bearing and the cylinders' chambers.

Each of the rotating pressure-sensing holes drilled in the shaft encounters a pressure spike once each revolution due to the always downward acting force of gravity and other unidirectional forces such as those created by wear or misalignment. This pressure spike is superimposed on the rotating pressure profile caused by unbalanced clothes and will cause unwanted motion of the pistons if not accounted for in the design. The hydraulic network, therefore, is designed with two components to render the pistons insensitive to pressure spikes. A capillary tube is built into each hydraulic line as a flow restrictor which prevents a rush of fluid through the lines because of the finite time required to force fluid through a certain size opening. It acts as a damper in the dynamic response of the system. Just ahead of the capillary tube, a section of flexible tubing is inserted to store the unwanted energy caused by the pressure spike. The tubing simply acts as a balloon, expanding and collapsing in response to large changes in pressure.

A schematic of the proposed hydraulic system is shown in Figure 14. The pressures on both sides of the piston are obtained by writing the flow equations between points A and D and points E and H.

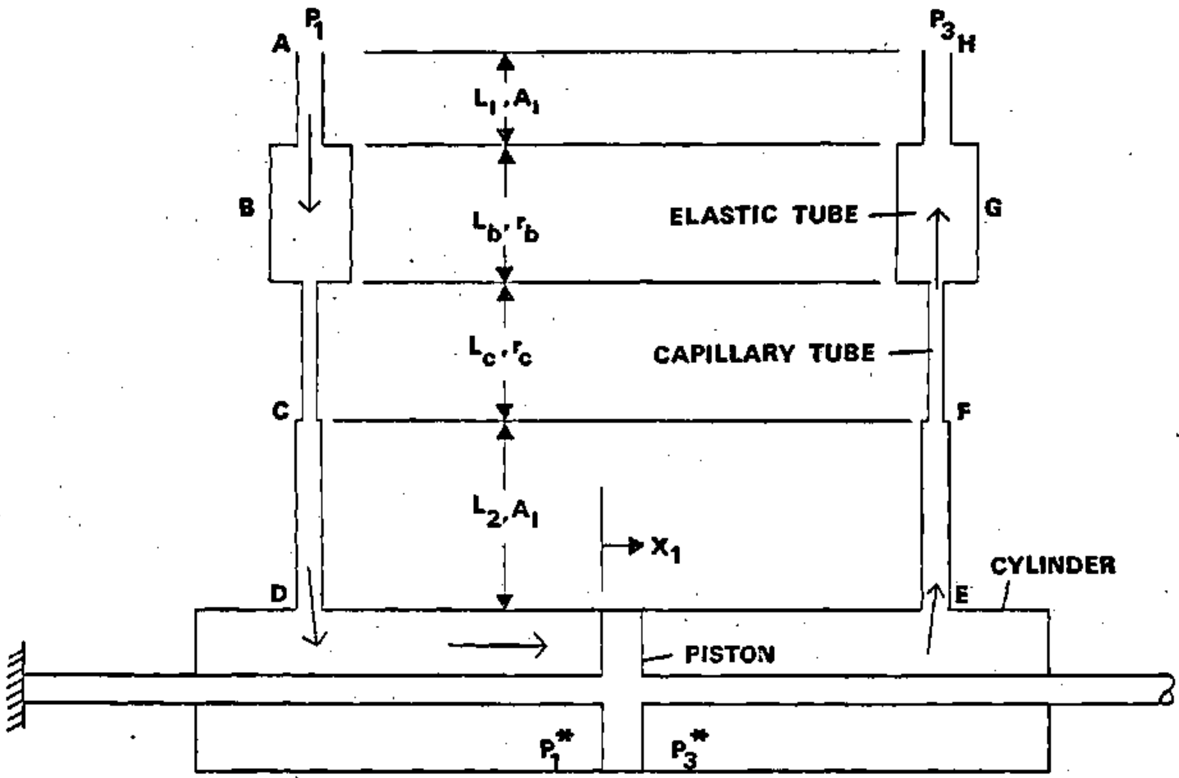

Figure 14. Schematic of the Proposed Hydraulic System

The equations for flow into the cylinder are different than those for flow out simply because the fluid encounters the elastic tube before the capillary tube going in and vice versa going out of the cylinder.

The assumption is made that there is an infinite supply of oil available at the pressure sources. Once the solution is obtained, of course, a check must be made to verify that the oil flow in the journal bearing is large enough to meet the predicted oil flow required in the cylinder. If it is not, the movement of the piston is altered and metal-to-metal contact can possibly occur in the bearing.

In writing the equations for flow, energy losses proportional to the velocity squared are neglected, greatly simplifying the flow equations. This can be justified by an order of magnitude analysis. The tube sections, of length L_1 and L_2 are designed such that laminar flow exists and the losses are primarily due to viscous forces and not to inertia forces. The energy loss due to inertia forces is of the form

$$h_i = \frac{V^2}{2g} \quad \text{(in.)}$$

while those due to viscous forces are

$$h_v = \frac{4LV^2}{2gd}\left(\frac{64}{Re}\right)$$

Typical values of these variables used in this analysis are:

$$L = 10 \text{ in.}$$

$$\mu = .00004 \text{ reyns}$$

$$V = .1 \text{ in/sec}$$

$$\gamma = .0311 \text{ lbf/in}^3$$

$$d = .0625 \text{ in}$$

therefore

$$h_v = \frac{(128)(10)(.1)(.00004)}{(.0311)(.0625)} = 2.65 \text{ in}$$

and

$$h_i = \frac{(.1)^2}{772} = .000013 \text{ in}$$

and the inertia losses can be neglected.

Oil is supplied to the openings in the shaft at A and H in Figure 14 at pressures P_1 and P_3, travels through the tubing, and provides pressures P_1^* and P_3^* acting on surface area A_s producing force to move the tub.

We assume P_1 is greater than P_3 and that flow is into chamber 1 and out of chamber 3. The continuity equation for an assumed incompressible fluid is

$$Q_1 = Q_{in} + Q_{r_1} \tag{19}$$

where

Q_1 = flow into tube at A, in^3/sec

Q_{in} = flow into chamber 1, in^3/sec

Q_{r_1} = flow in radial direction at elastic tube, in^3/sec

Flow from point A into chamber 1 only is described by

$$\frac{P_1}{\gamma} = \frac{P_1^*}{\gamma} + C_1 V_{AB} + C_2 \frac{dX_1}{dt} + C_3 V_{CD} \tag{20}$$

where

$$C_1 = \frac{128\mu L_1}{\gamma d_1^2} \text{ loss constant in line AB, sec} \tag{21}$$

$$C_2 = \frac{128\mu L_c}{\gamma d_c^4} \text{ loss constant in capillary, sec} \tag{22}$$

$$C_3 = \frac{128\mu L_2}{\gamma d_2^2} \text{ loss constant in line CD, sec} \tag{23}$$

V_{AB} = velocity of fluid flowing in line AB, in/sec

$V_{CD} = \frac{A_s}{A_1} \frac{dX_1}{dt}$ = velocity of the fluid in line CD, in/sec

Flow from point A through the elastic tube is governed by

$$\frac{P_1}{\gamma} = \frac{P_B}{\gamma} + C_1 V_{AB} \tag{24}$$

Two of the terms in equation (19) can be rewritten as

$$Q_1 = A_1 V_{AB}$$
$$Q_{in} = A_s \frac{dX_1}{dt} \tag{25}$$

We now need an expression for the radial flow in the elastic tube. Neglecting end effects in the elastic tube, the volume of a cylinder is

$$V_c = \pi r_b^2 L_b$$

and the radial flow rate is

$$Q_r = \frac{dV_c}{dt} = 2\pi r_b L_b \tag{26}$$

If r_i is the initial inner radius and u is the radial displacement of the elastic tube, then

$$r_b = r_i + u$$

and therefore

$$\frac{dr_b}{dt} = \frac{d}{dt}(r_i+u) = \frac{du}{dt}$$

so equation (26) becomes

$$Q_r = 2\pi L_b(u+r_i)\frac{du}{dt} \tag{27}$$

From strength of materials analysis [7],

$$u = r_i\epsilon_\theta \tag{28}$$

and

$$\epsilon_\theta = \frac{1}{E}\left[(1-\nu^2)\sigma_\theta - \nu(1+\nu)\sigma_r\right] \tag{29}$$

For a thin-walled cylinder,

$$\sigma_\theta \approx \frac{P_B(r_b)}{t} = \frac{P_B}{t}(r_i+u) \tag{30}$$

$$\sigma_r \approx 0$$

If $u << r_i$ equation (30) becomes

$$\sigma_\theta = \frac{P_B}{t} r_i \tag{31}$$

This assumption is justified by an order of magnitude analysis of the terms. A pressure of 100 psi produces a radial displacement of approximately .00005 inches in the elastic tube used in this analysis with an inside diameter of .1375. Thus $u << r_i$ and equation (31) is a good approximation to the true stress.

Substituting equation (31) into equation (28) gives

$$u = r_i \epsilon_\theta = \frac{r_i}{E}(1-\nu^2)\frac{P_B}{t} r_i \tag{32}$$

or

$$u = C_6 P_B$$

where

$$C_6 = \frac{r_i^2(1-\nu^2)}{Et}, \quad \frac{in^3}{lbf}$$

and

$$\frac{du}{dt} = C_6 \frac{dP_B}{dt}$$

so the expression for radial flow in the elastic tube subjected to internal pressure P_B is

$$Q_r = 2\pi \ell_b C_6 (C_6 P_B + r_b) \frac{dP_B}{dt} \tag{33}$$

Substituting this expression for the radial elastic tube flow into equation (19), we get for the flow equations

$$A_1 V_{AB} = 2\pi L_b C_6 r_b \frac{dP_B}{dt} + 2\pi L_b C_6^2 P_B \frac{dP_B}{dt} + A_s \frac{dX_1}{dt} \tag{34}$$

$$\frac{P_1}{\gamma} = \frac{P_1^*}{\gamma} + C_1 V_{AB} + C_2 \frac{dX_1}{dt} + C_3 \frac{A_s}{A_1} \frac{dX_1}{dt} \tag{35}$$

$$\frac{P_1}{\gamma} = \frac{P_B}{\gamma} + C_1 V_{AB} \tag{36}$$

To summarize, equation (34) is the flow continuity equation, while equations (35) and (36) describe the pressure drop in the hydraulic lines for flow into the cylinder only and flow into the elastic tube only, respectively. These are three equations in three unknowns, V_{AB}, P_1^*, and P_B, if we assume $\frac{dP_B}{dt}$ to be known, an assumption later justified.

Solving for P_1^*, the pressure on the left face of the piston, we get

$$P_1^* = \frac{\dfrac{A_1 P_1}{\gamma} - 2\pi L_b C_1 C_6 r_b \dfrac{dP_B}{dt} - \dfrac{dX_1}{dt}\left[A_1\left(C_2 + C_3 \dfrac{A_s}{A_1}\right) + 2\pi L_b \gamma \dfrac{dP_B}{dt} C_1 (C_2 + C_3) + C_1 A_s\right]}{\dfrac{A_1}{\gamma} + 2\pi L_b C_1 C_6^2 \dfrac{dP_B}{dt}} \tag{37}$$

The pressure on the opposite piston surface is found in a similar manner. The flow continuity equation, assuming flow out of cylinder chamber 3 is

$$Q_{out} = Q_3 + Q_{r_2} \tag{38}$$

and

$$Q_{out} = Q_{in} = A_s \frac{dX_1}{dt} = \text{flow leaving chamber 3, } \ \text{in}^3/\text{sec}$$

$$Q_3 = A_1 V_{GH} = \text{flow out of line at H, } \ \text{in}^3/\text{sec}$$

$$Q_{r_2} = 2\pi L_b (C_6 P_G + r_b) C_6 \frac{dP_G}{dt}$$

For flow from the cylinder to the elastic tube only,

$$\frac{P_3^*}{\gamma} = \frac{P_G}{\gamma} + \frac{A_s}{A_1} C_3 \frac{dX_1}{dt} + C_2 \frac{dX_1}{dt} \tag{39}$$

where P_G = pressure at elastic tube at G in Figure 12, psi. The extra term exists because flow going out must pass through the capillary tube first. The three simultaneous equations describing flow out of the cylinder are thus

$$A_s \frac{dX_1}{dt} = 2\pi L_b C_6^2 P_G \frac{dP_G}{dt} + 2\pi L_b r_b C_6 \frac{dP_G}{dt} + A_1 V_{GH} \tag{40}$$

$$\frac{P_3^*}{\gamma} = \frac{P_3}{\gamma} + C_3 \frac{A_s}{A_1} \frac{dX_1}{dt} + C_1 V_{GH} + C_2 \frac{dX_1}{dt} \tag{41}$$

$$\frac{P_3^*}{\gamma} = \frac{P_G}{\gamma} + C_3 \frac{A_s}{A_1} \frac{dX_1}{dt} + C_2 \frac{dX_1}{dt} \tag{42}$$

with P_3^*, P_G, and V_{GH} the three unknowns, again assuming that $\frac{dP_G}{dt}$, the time derivative of P_G, is known.

Solving for P_3^*, the pressure on the right side of the piston, we get

$$P_3^* = \frac{\frac{A_1 P_3}{\gamma} + 2\pi L_b r_b C_1 C_6 \frac{dP_G}{dt} + \frac{dX_1}{dt}\left[A_1(C_2+C_3) + C_1 A_s + 2\pi C_1 \gamma L_b C_6^2 \left(C_2 + \frac{A_s}{A_1} C_3\right) \frac{dP_G}{dt}\right]}{\frac{A_1}{\gamma} + 2\pi L_b C_1 C_6^2 \frac{dP_G}{dt}} \tag{43}$$

For the second cylinder with pressures P_2 and P_4 applied at the sensor bearing, a similar analysis provides the following expressions for the pressures P_2^* and P_4^* on each side of the piston:

$$P_2^* = \frac{\frac{A_1 P_2}{\gamma} - 2\pi L_b r_b C_1 C_6 \frac{dP_B}{dt} - \frac{dX_2}{dt}\left[A_1\left(C_2 + C_3 \frac{A_s}{A_1}\right) + C_1 A_s + 2\pi L_b \gamma C_6^2 \frac{dP_B}{dt} C_1 (C_2 + C_4)\right]}{\frac{A_1}{\gamma} + 2\pi L_b C_1 C_6^2 \frac{dP_B}{dt}} \tag{44}$$

$$P_4^* = \frac{\dfrac{A_1 P_4}{\gamma} - 2\pi L_b r_b C_1 C_6 \dfrac{dP_G}{dt} + \dfrac{dX_2}{dt}\left[A_1(C_2+C_3)+C_1 A_s + 2\pi L_b \gamma C_1 C_6^2 \left(C_2 + C_3 \dfrac{A_s}{A_1}\right)\dfrac{dP_G}{dt}\right]}{\dfrac{A_1}{\gamma} + 2\pi L_b C_1 C_6^2 \dfrac{dP_G}{dt}}$$

Now we have expressions for the pressures on the piston surfaces to be used in the equations of motion of the pistons, equations (3) and (4).

As mentioned before the expressions for pressure are valid only if $\frac{dP_B}{dt}$ is known. The physical significance of this term in the pressure expressions lies in its relationship with the elastic tube. If $\frac{dP_B}{dt}$ is zero, the elastic tube has a fixed inner radius and is not expanding or contracting. Referring to equation (24), we take the time derivative of both sides of the equation and get

$$\frac{d}{dt} P_B = \frac{d}{dt} P_1 + C_1 \frac{d}{dt} V_{AB} \tag{46}$$

Since $\frac{dP_1}{dt}$ is much greater than $\frac{dV_{AB}}{dt}$, as would be the case if a sudden pressure spike were to appear at the end of the hydraulic line, then $\frac{dP_B}{dt}$ is approximately equal to $\frac{d}{dt} P_1$. It is thus assumed that the time derivative of the pressures at the elastic tubes on each hydraulic line is equal to the time derivative of the pressure at the end of the hydraulic line.

Solution of Equations

The two second order equations describing the motion of the pistons are solved simultaneously using the Runge-Kutta method [8], by which the increments of the functions are calculated all at once by means of a prescribed set of equations. An advantage of this method is that the calculations for the first increment are identical to the calculations of any other increment, thus simplifying the computer program. Also the interval of the independent variable time can be changed at any instant during the calculations.

Equations (3) and (4) to be solved are repeated here:

$$\frac{d^2X_1}{dt^2} = (P_1^*-P_3^*)A_s - FX_1$$

$$\frac{d^2X_2}{dt^2} = (P_2^*-P_4^*)A_s - FX_2$$

This system of equations can be reduced to four first order equations by setting

$$Y_1 = \frac{dX_1}{dt} \quad \text{and} \quad Y_2 = \frac{dX_2}{dt} \tag{47}$$

The four differential equations to be solved simultaneously are then

$$\frac{dY_1}{dt} = (P_1^*-P_3^*)A_s - FX_1 = f_1[t,X_1(i),Y_1(i),X_2(i),Y_2(i)] \tag{48}$$

$$\frac{dX_1}{dt} = Y_1 = f_2[Y_1(i)] \tag{49}$$

$$\frac{dY_2}{dt} = (P_2^*-P_4^*)A_s - FX_2 = f_3[t,X_1(i),Y_1(i),X_2(i),Y_2(i)] \qquad (50)$$

$$\frac{dX_2}{dt} = Y_2 = f_4[Y_2(i)] \qquad (51)$$

where i is the site for which the values are desired.

Four initial conditions are needed; these depend on the initial position of the tub. Since the machine starts from rest, the initial velocity of the pistons is zero. The integration is carried out step by step by means of the following formulas in the order given, with h_t being the desired time interval:

$$\begin{aligned} K_{11} &= f_1[Y_1(i)] \\ K_{21} &= f_2[t,X_1(i),Y_1(i),X_2(i),Y_2(i)] \\ K_{31} &= f_3[Y_2(i)] \\ K_{41} &= f_4[t,X_1(i),Y_1(i),X_2(i),Y_2(i)] \end{aligned} \qquad (52)$$

$$\begin{aligned} K_{12} &= f_1[Y_1(i)+h_tK_{21}/2] \\ K_{22} &= f_2[t+h_t/2,X_1(i)+h_tK_{11}/2,Y_1(i)+h_tK_{21}/2, \\ &\qquad X_2(i)+h_tK_{31}/2,Y_2(i)+h_tK_{41}/2] \\ K_{32} &= f_3[Y_2(i)+h_tK_{41}/2] \end{aligned} \qquad (53)$$

$$K_{42} = f_4[t+h_t/2, X_1(i)+h_tK_{11}/2, Y_1(i)+h_tK_{21}/2,$$
$$X_2(i)+h_tK_{31}/2, Y_2(i)+h_tK_{41}/2]$$

$$K_{13} = f_1[Y_1(i)+hK_{22}/2]$$

$$K_{23} = f_2[t+h_t/2, X_1(i)+h_tK_{12}/2, Y_1(i)+h_tK_{22}/2,$$
$$X_2(i)+h_tK_{32}/2, Y_2(i)+h_tK_{42}/2]$$

$$K_{33} = f_3[Y_2(i)+h_tK_{42}/2] \tag{54}$$

$$K_{43} = f_4[t+h_t/2, X_1(i)+h_tK_{12}/2, Y_1(i)+h_tK_{22}/2,$$
$$X_2(i)+h_tK_{32}/2, Y_2(i)+h_tK_{42}/2]$$

$$K_{14} = f_1[Y_1(i)+h_tK_{23}]$$

$$K_{24} = f_2[t+h_t, X_1(i)+h_tK_{13}, Y_1(i)+h_tK_{23},$$
$$X_2(i)+h_tK_{33}, Y_2(i)+h_tK_{43}]$$

$$K_{34} = f_3[Y_2(i)+h_tK_{43}] \tag{55}$$

$$K_{44} = f_4[t+h_t, X_1(i)+h_tK_{13}, Y_1(i)+h_tK_{23},$$
$$X_2(i)+h_tK_{33}, Y_2(i)+h_tK_{43}]$$

and the values for each interval are found by

$$X_1(i+1) = X_1(i) + h_t(K_{11}+2K_{12}+2K_{13}+K_{14})/6$$

$$Y_1(i+1) = Y_1(i) + h_t(K_{21}+2K_{22}+2K_{23}+K_{24})/6$$

$$X_2(i+1) = X_2(i) + h_t(K_{31}+2K_{32}+2K_{33}+K_{43})/6 \qquad (56)$$

$$Y_2(i+1) = Y_2(i) + h_t(K_{41}+2K_{42}+2K_{43}+K_{44})/6$$

The values for each succeeding interval are computed in exactly the same manner, i.e., using the values of the functions at the previous interval.

Computer Program

The computer program serves to perform the numerical calculations that predict the motion of the tub and is the principal tool for the optimization of the design parameters. The total program consists of a main program and eight subprograms that provide data about different phases of the system operation. The program is written in FORTRAN IV language and is included in the appendix.

The primary purpose of the main program is to perform the steps in the Runge-Kutta numerical method of solving the differential equations of motion. It also reads in the data, sets initial conditions, and prints the output. In this program are determined the displacement and velocity of each piston and the oil flow in the journal bearing for each interval of time.

The other subprograms individually determine the total inertia force, the force on the journal bearing, the eccentricity ratio, the load per unit area carried by the journal bearing, the attitude angle, and the pressures developed at the holes in the shaft at each time interval.

CHAPTER III

ANALYSIS OF RESULTS

Introduction

The purpose of the analytical stage of research is the implementation of the data obtained from the analysis into results that guide future researchers into the actual construction of a physical model of the proposed system. This chapter discusses the important design parameters in the system and attempts to categorize those which have the most effect on the operation and efficiency of the balancing system. Because of the large number of parameters which affect more than one phase of the system operation, an attempt is made to find an optimum range of the parameters rather than find the "ideal machine" through this analysis alone. On the basis of data obtained by parameter variation, it was concluded that certain of these parameters could be narrowed to precise values, while several combinations of the other parameters were found to give similar results.

The graphs on the following pages represent particular solutions obtained for a combination of parameters that were found to satisfactorily meet the requirements for efficient system operation and justify the assumptions made earlier in the analysis. The data are presented in this manner because they are typical of data obtained for any combination of parameters that produce satisfactory performance of the balancing system. Finally on the basis of conclusions reached in the analysis, recommendations are made for future research.

Discussion and Conclusions

The important parameters which affect the operation of the system are

1. Viscosity of the hydraulic fluid-lubricant
2. Angular acceleration of the tub
3. Distance between support bearings
4. Length/diameter ratio of the tubing in the hydraulic lines
5. Surface area of the pistons
6. Initial radial clearance between the shaft and journal bearing
7. Total weight of the tub.

As mentioned before, variation of any of the above parameters affects more than one part of the system operation. For example, an increase in the viscosity of the fluid increases the losses in the hydraulic lines but decreases the operating eccentricity ratio, thereby decreasing the pressure developed in the bearing; a decrease in machine angular acceleration decreases the maximum inertia force, but increases the time for balance; an increase in the radial clearance between the shaft and bearing increases the amount of oil flowing in the bearing but also reduces the pressures generated in the bearing. Other parameters have similar multiple effects on the system.

The parameters remaining constant during the analysis are

1. The weight of the unbalanced clothes = 6 lbf

2. The radius of the unbalanced clothes from the tub center = 11 inches

3. Time interval in numerical calculations = .05 seconds

These machine parameters represent a worst case for balance in the washing machine. Any values below this will reduce the amount of tub motion needed for balance.

The data following is for various positions of unbalance with the machine parameters fixed. Because of the symmetry of the tub about a radial line 45 degrees between the cylinders, only balance locations of θ = 0, 15, 30, and 45 degrees were tested. Variation of these parameters alter the path of tub motion, time for balance, and maximum inertia force experienced. For these solutions,

$$\mu = .00006 \text{ reyns} = 414 \text{ centipoise}$$

$$\text{Machine Acceleration} = 1100 \text{ rev/min}^2 = 1.93 \text{ rad/sec}^2$$

$$B_s = 3.9 \text{ inches}$$

$$L_1 = 20 \text{ inches}$$

$$D_1 = .125 \text{ inches}$$

$$L_2 = 20 \text{ inches}$$

$$D_2 = .125 \text{ inches}$$

$$L_c = 1.5 \text{ inches}$$

$$A_s = .2 \text{ in}^2$$

$$c = .00006 \text{ in}$$

Weight of Tub, Clothes, and Water = 70 lbf

Figures 15-18 show the motion of the tub center with respect to the shaft center for unbalanced clothes at the four test locations of θ. The fact that the pressure profile is not symmetrical about any radial line from the shaft center (see Figure 11) accounts for the nonlinear motion of the tub center toward its position of balance in each of the graphs. The tub will always seek this point of balance, however, because the further it moves away from this equilibrium position, the larger the inertia force and the larger the pressures generated to move the tub toward the balance position. Once this position is reached, the inertia force disappears as does the rotating pressure profile, and equilibrium is reached.

The maximum pressure generated versus the machine speed is shown in Figure 19 for unbalance at $\theta = 45$ degrees. Typical of the profile for all positions of θ, the generated pressure goes to zero as the tub is balanced.

Figure 20 displays inertia force versus machine speed and time for $\theta = 45$ degrees. In this graph the most critical point to notice is the maximum inertia force that must be dealt with. This must be known so that a suspension system can be designed to properly handle this force transmitted to the machine supports.

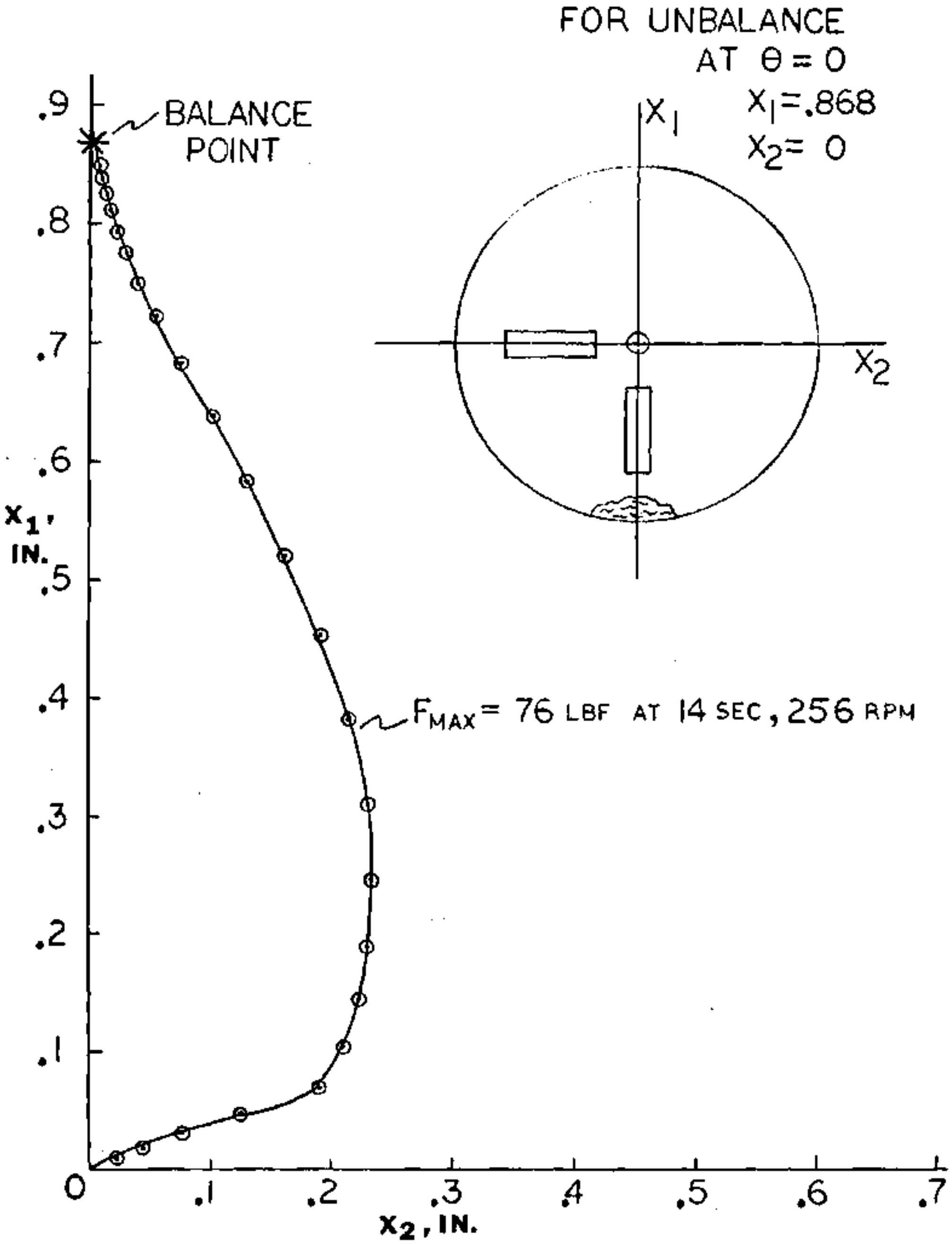

Figure 15. Motion of Tub Relative to Shaft Center for Unbalance at θ = 0 Degrees

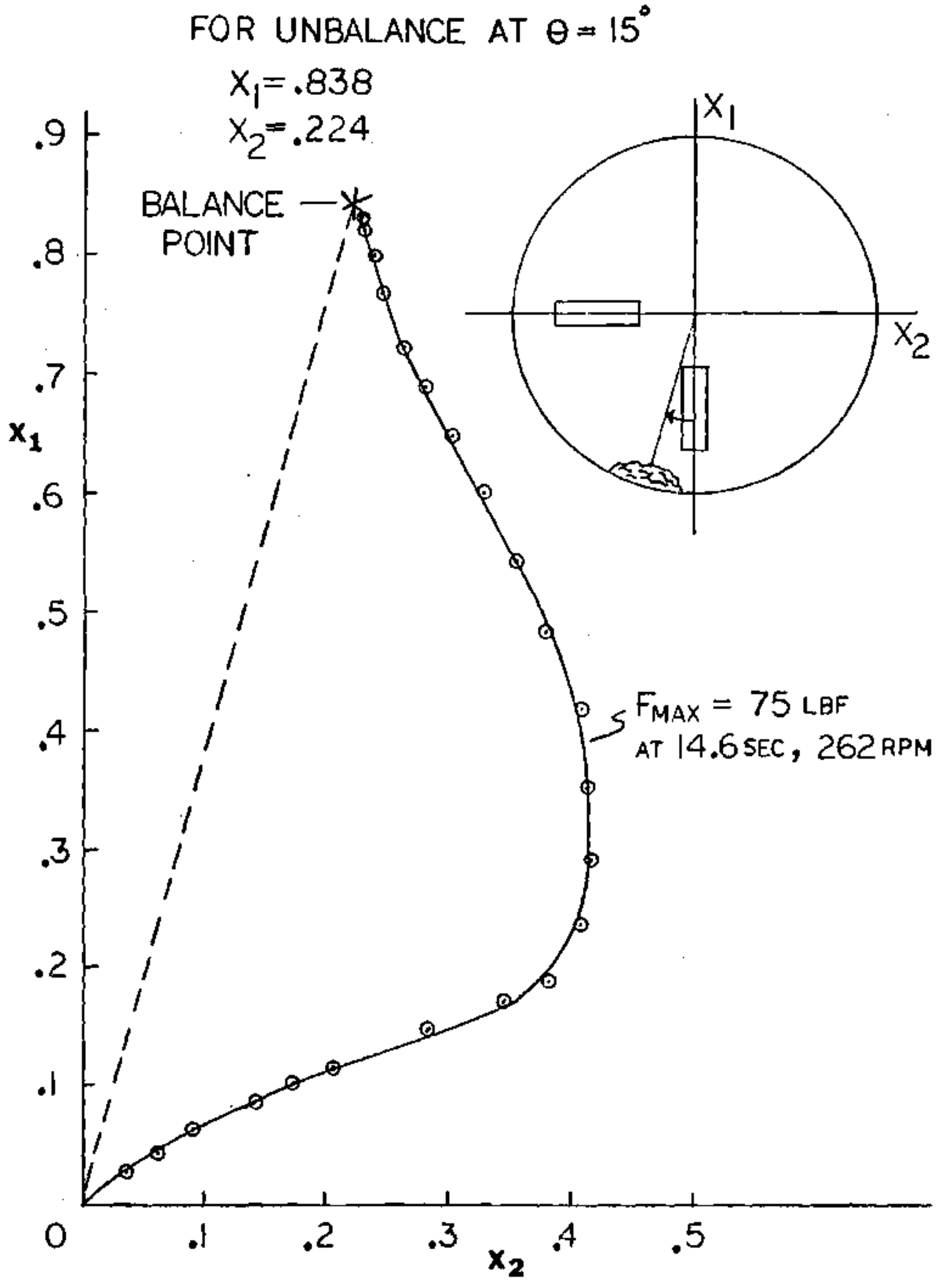

Figure 16. Motion of Tub Relative to Shaft Center for Unbalance at θ = 15 Degrees

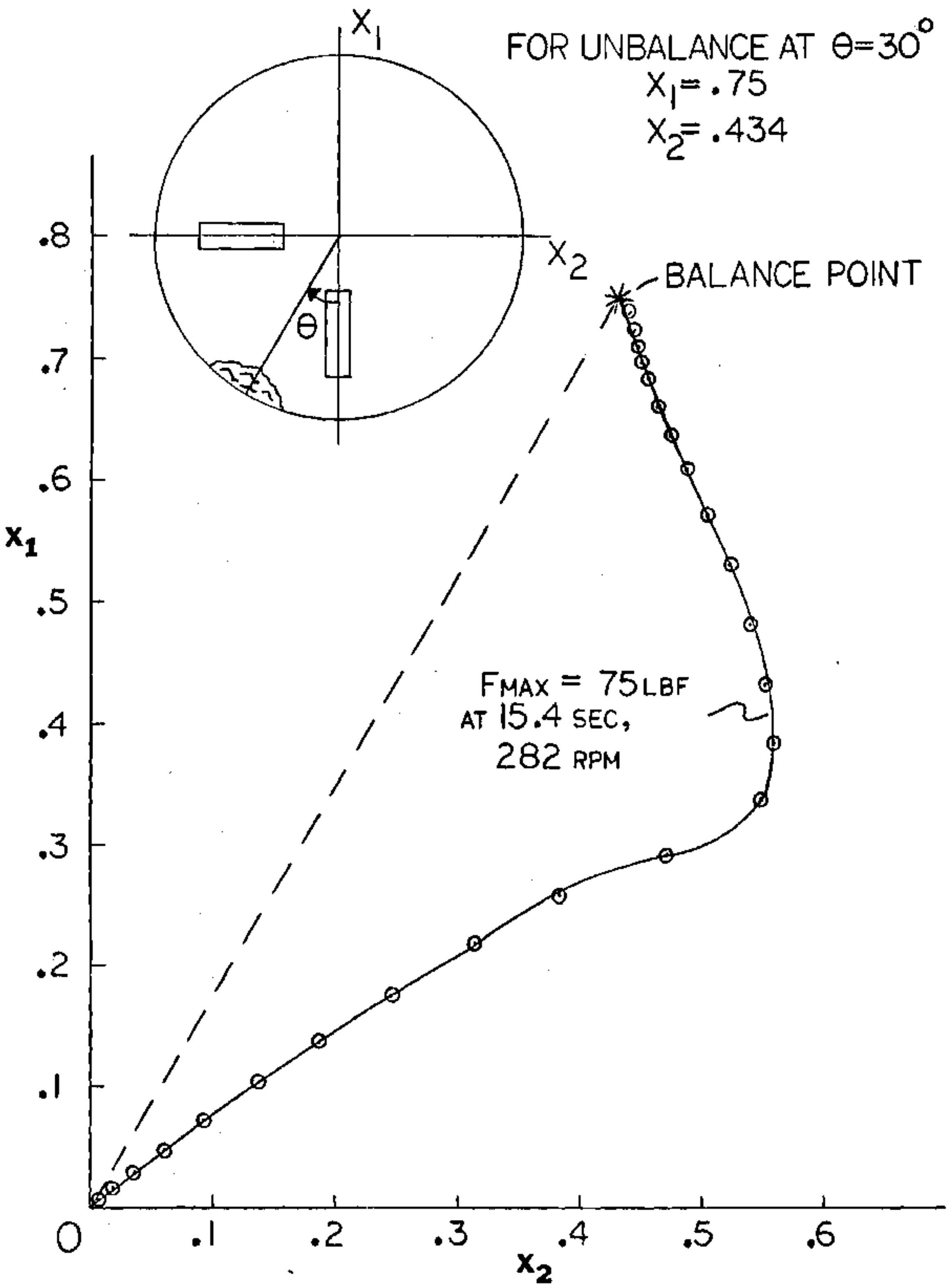

Figure 17. Motion of Tub Relative to Shaft Center for Unbalance at θ = 30 Degrees

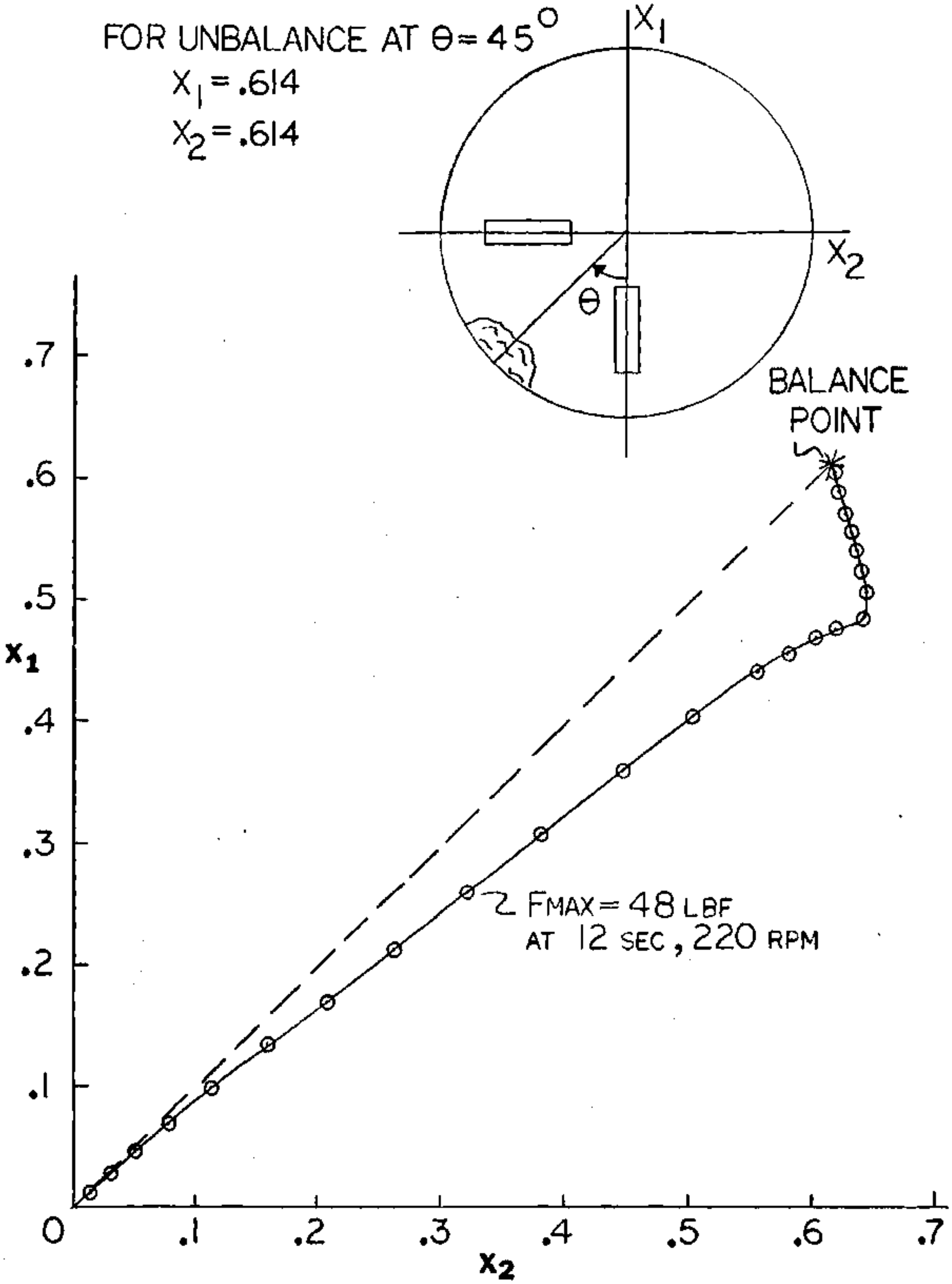

Figure 18. Motion of Tub Relative to Shaft Center for Unbalance at θ = 45 Degrees

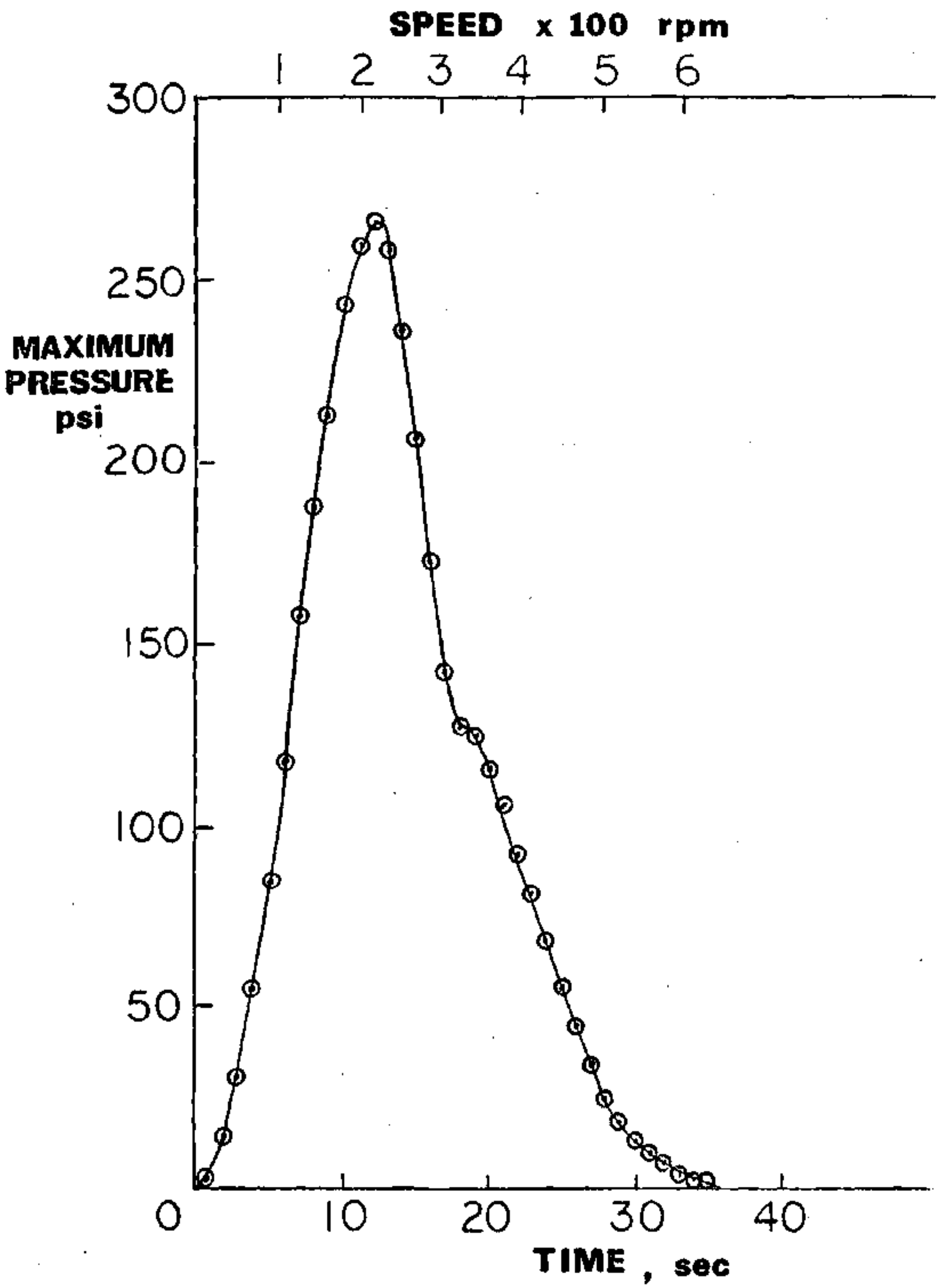

Figure 19. Maximum Pressure Generated in Journal Bearing for Unbalance at θ = 45 Degrees

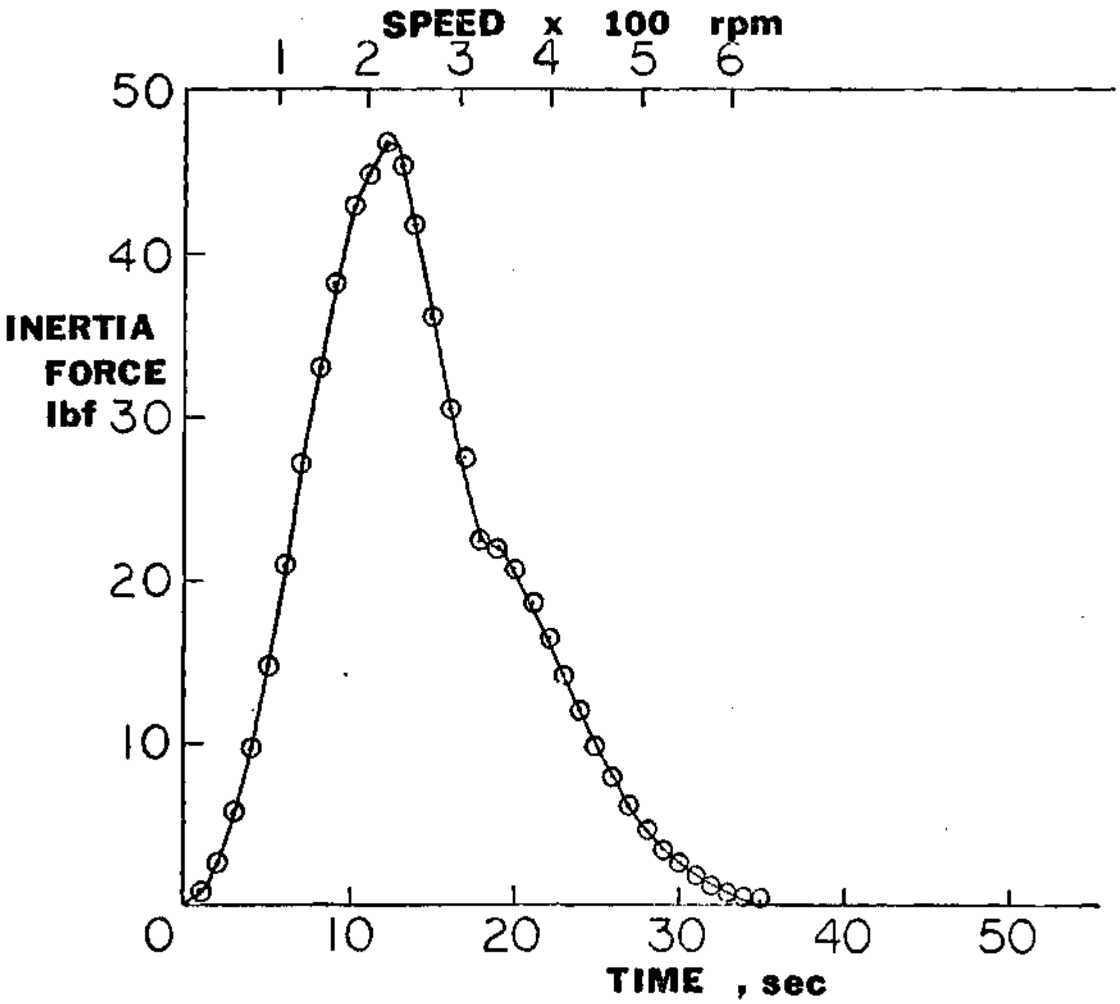

Figure 20. Inertia Force for Unbalance at θ = 45
Degrees Versus Tub Speed and Time

Figure 21 shows the oil flow required in both cylinders and the oil flow available in the journal bearing versus machine speed and time. If the oil flow predicted in both cylinders exceeds that available in the bearing, the pistons will not move as predicted because of a lack of oil exerting pressure on the pistons' faces. The combination of all other parameters chosen for a particular test must insure an adequate supply of oil for all magnitudes and locations of the inertia force and at all times during the balancing process. Since we assume an adequate supply of oil in the mathematical prediction of the pistons' motion, proper operation of the system depends on the total oil flow at both pistons not exceeding the amount of oil available at the journal bearing. In the range of loads and speed in this analysis, previous solutions to journal bearing operating equations predict that generally less than 10 per cent of the oil flowing in the bearing is lost to side leakage [4]. The rest is available to be used for flow to the pistons.

The early test results indicated a requirement for oil greater than available at the bearing. Efforts were thus made to either increase the amount of oil available at the bearing or to decrease the amount needed at the pistons.

The range of bearing operating conditions in this research, described by the dimensionless Sommerfeld number S, is $1 < S < 5000$. The corresponding range of the dimensionless flow variable is $3.4 < Q/rc\omega L < 3.2$, and is more often near 3.4. For a journal bearing of fixed dimensions r, l, and c, the flow in the bearing is

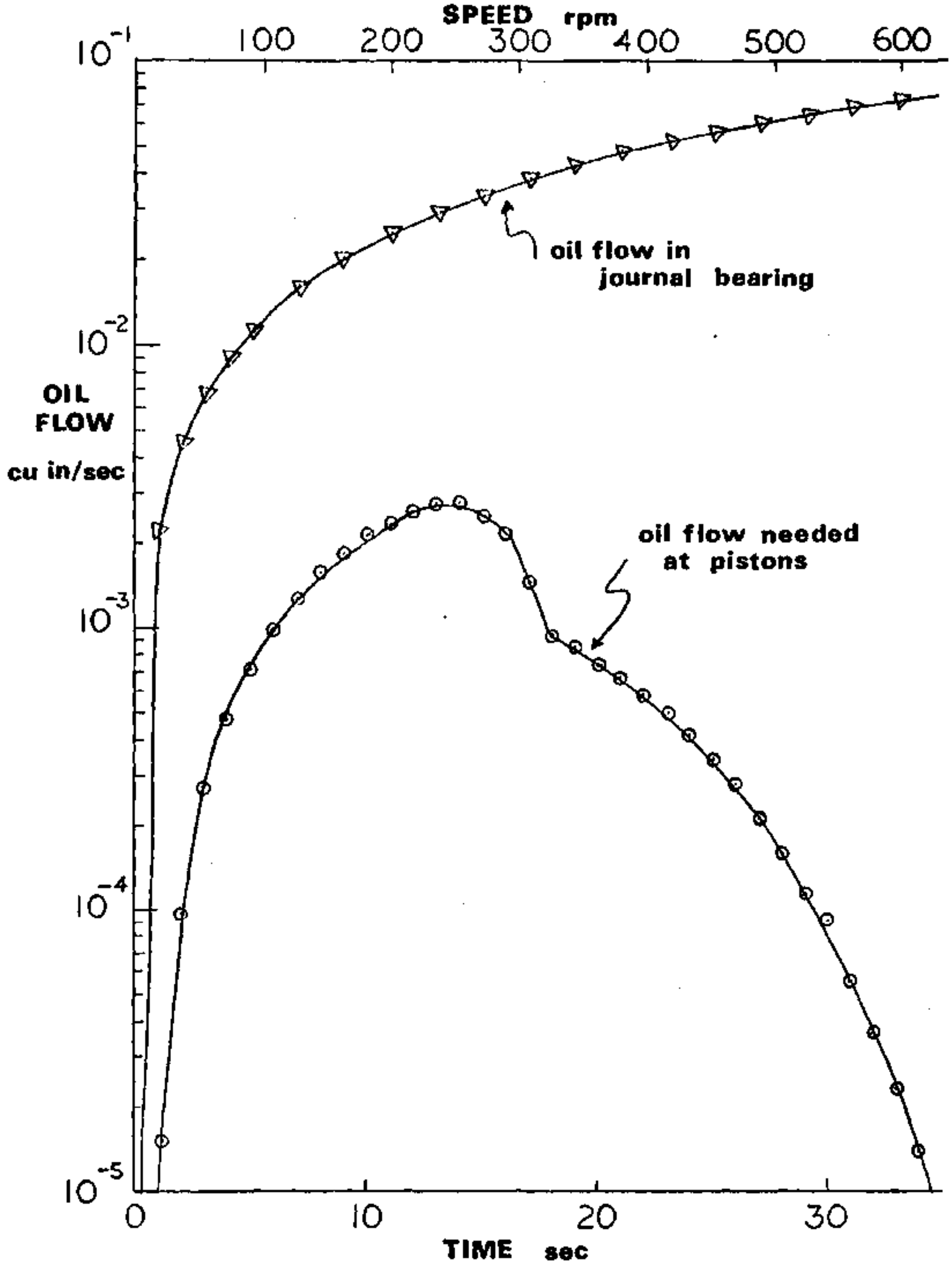

Figure 21. Comparison of Oil Flow Available in Journal Bearing with Oil Flow Needed at Pistons for Unbalance at θ = 45 Degrees

$$Q = K\omega \tag{57}$$

or approximately linearly proportional to the machine speed, since the range of K is $3.4\ rcL < K < 3.2\ rcL$. Hence the amount of oil available depends primarily on the shaft speed and is relatively independent of the other parameters in the balancing system. Since the oil flow needed at each piston is the product of the surface area of the piston times its velocity , the flow required at the pistons can be reduced by causing the pistons to move at as slow a velocity as possible and still allow them to move in the proper direction. The forces assumed to be acting on each piston in this analysis are (1) an inertia force, proportional to the machine speed squared and (2) a force due to the pressure difference acting on the pistons. The velocity of the pistons can be reduced by having the force due to the pressure difference exceed the inertia force by as small an amount as possible during the balancing operation. Each of the parameters listed at the beginning of this section affect the pressure difference encountered at the pistons.

The location of the journal bearing relative to the other support(s) determines the portion of the total inertia force it will carry. This affects the operation of the pistons simply because the larger the share of the load carried by the journal bearing, the larger the pressures generated for a given inertia force and machine speed and vice versa. If the journal bearing carries too large a share of the load, the control system will be too sensitive and will cause a depletion of the oil supply available in the journal bearing. If, however, the bearing is not sensitive enough, the pressures generated might not be large enough to oppose the inertia force on each piston--a condition which will lead to piston motion in the wrong direction for balance.

The energy lost during the flow of fluid from the journal bearing to the pistons is a function of the dimensions of the tubing in the hydraulic lines, the viscosity of the fluid, and the velocity of the pistons. If these losses are too large the pressure of the fluid on the pistons' surfaces will be too small to move them in the proper direction. Conversely, losses too small may cause a depletion of the supply of oil, since the pistons will try to move faster than oil can be supplied to them. Since the losses in the hydraulic line serve as a damper in the dynamic response of the system, losses too small can also cause unwanted oscillation about the position of balance.

The choice of fluid for the system is complicated by its two-fold purpose: (1) for its properties in the journal bearing, i.e., load supporting capacity, ability to remove heat from the system, rust and corrosion preventative properties, chemical stability, and others; (2) its function of transferring fluid from the journal bearing to the pistons. Ideally, we would probably select two different fluids--one for each of the above tasks. The choice of the fluid must be based, therefore, on its ability to perform satisfactorily in both capacities. The final value of viscosity chosen must be obtained by predicting an operating temperature. This would require an iterative procedure since the temperature is a function of the viscosity.

Rather than specifying exact length/diameter ratios for the tubing in the hydraulic lines, we can most simply discuss the effect of the flow through these lines on the system response by the definition of a new variable τ, the loss parameter, where

$$\tau = \gamma \left[C_2 + (C_1 + C_3) \frac{A_s}{A_1} \right] \quad \text{(sec)} \tag{58}$$

and the terms C_1, C_2, and C_3 are defined in equations (21), (22), and (23). Neglecting energy losses in the elastic tube, this expression multiplied by the velocity of the piston in each cylinder gives the energy loss of the fluid in inches in the incoming lines at each cylinder. We can neglect the losses due to the expansion of the elastic tube because they are much smaller than the viscous flow losses. The constants C_1, C_2, and C_3 contain among other terms, the dimensions of the tubing in the lines, and for a chosen fluid it is thus not necessary to specify dimensions for this tubing, but rather we can specify τ for the configuration. For example, the effect of increasing the length of line one alone is the same as decreasing the diameter of any of the other incoming lines by a proportional amount. If the experimenter desires to test a certain system, he can vary τ with only a valve, rather than rebuilding the hydraulic line.

The data obtained showed that balance most often occurred without exceeding the available oil supply in the general range $240 < \tau < 330$. Losses much larger than this generally led to piston motion in the wrong direction, and balance did not occur. For small values of τ, piston velocities depleting the available oil were predicted.

The surface area of the pistons is important since this determines the force on the pistons for a given pressure. Too large an area can cause excessive piston velocity depleting the oil supply. Because of the restriction of piston velocities, the surface area of the pistons (total piston area minus area of piston rods) is restricted to rather small values. For the solutions previously presented the area is .2 in^2 with the distance between the support bearings is 3.9 inches. The combination of these two parameters is important since an increase in the surface area has the opposite effect as an increase in the distance between bearings. A larger surface area increases the force on the piston, while an increase in the distance between bearings decreases the pressures generated in the bearing. The designer is left with several choices of these two parameters which will allow balance to occur. The surface area of the pistons A_s, however, also affects the pressures on the piston faces since it determines the energy losses in the hydraulic lines (see equation (58)).

It was found that the maximum allowable velocity of either piston was below .1 in/sec in nearly all cases. The penalty for having the balancing system operate this slowly is the size of the inertia force that must be tolerated. For a machine with constant angular acceleration the inertia force increases as the angular velocity squared while the pistons move slowly to the balance position. This, of course, can be altered by a change in the machine acceleration, or even the use of a "stutter speed" at which the tub revolves at a constant velocity while the pistons are allowed to "catch up" with the inertia force. The effect of machine angular acceleration on the maximum inertia force is shown in Figure 22 for unbalance at $\theta = 45$ degrees and the machine parameters as specified in the previous solutions. Fortunately, this maximum inertia force need be tolerated for only a short time period after which it vanishes as balance occurs. An increase in machine angular acceleration alone will increase the oil flow and pressures generated, but will also increase the inertia force over the same time period.

An increase in the total weight of the tub is helpful since this decreases the distance required to balance a given unbalance. This can be seen as an easily implemented improvement in the current machine since the mere addition of the balancing mechanism to the tub would increase the weight. Problems might exist if the total load of clothes and water was very light, since it compromises about one-fourth of the weight of the tub, and the tub motion needed to balance a six-pound inertia force would be restrictive. During the spin cycle, about 12 per cent of the water is removed [3], also reducing the total tub weight. The tub will move further outward to compensate for this reduction in tub weight.

The clearance between the shaft and the journal bearing is important for several reasons. If too large, the pressures generated will be too small to balance the tub; if too small, there will not be enough oil available in the bearing to move the pistons. Machining

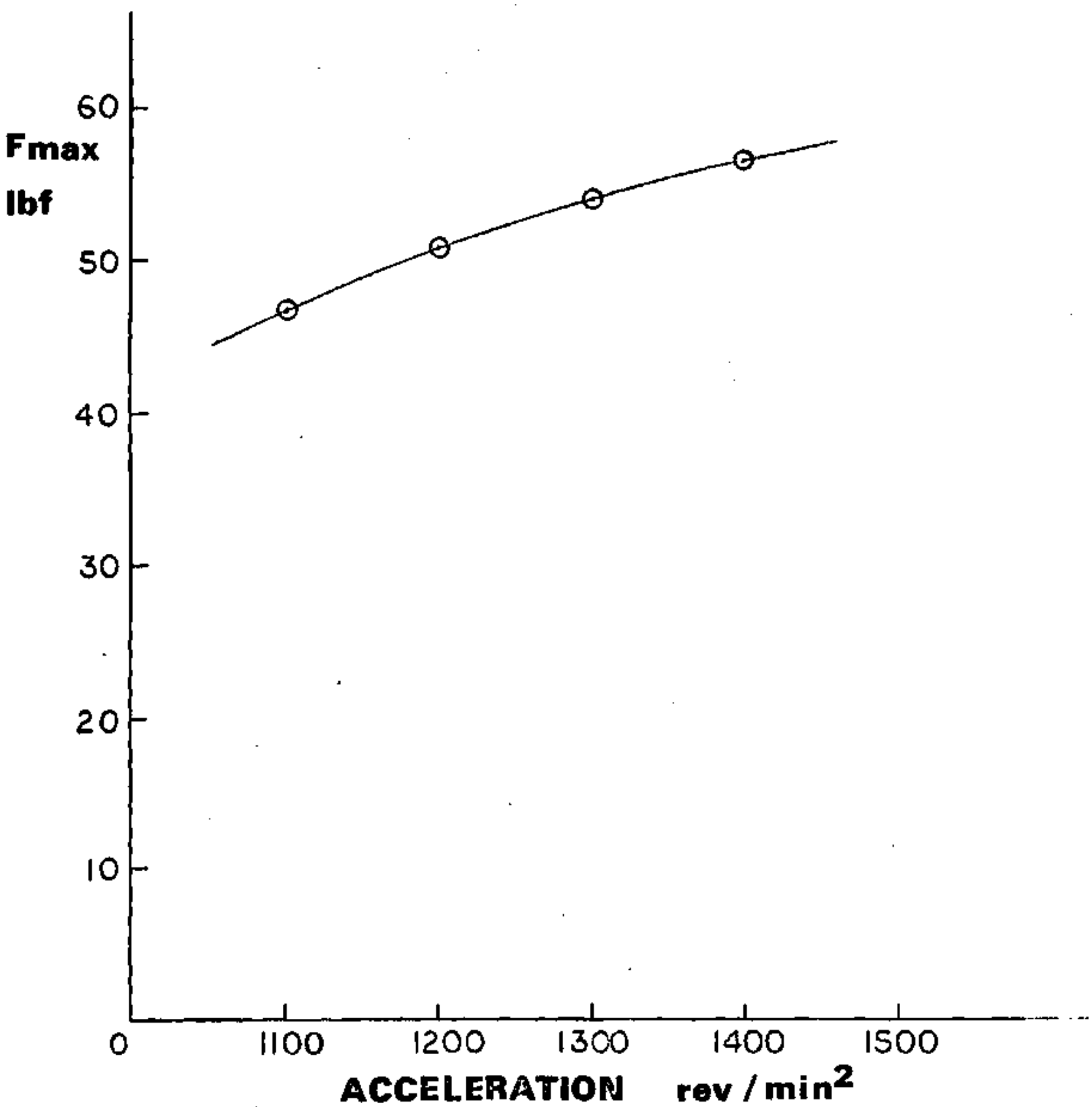

Figure 22. Effect of Variation of Machine Acceleration on Maximum Inertia Force During Balancing Process for Unbalance at θ = 45 Degrees

costs can also restrict the size of the clearance. A clearance of .0006 inches proved to be adequate over a wide range of other parameters. Clearances much larger than this proved to generate pressures too small to move the tub in the proper direction.

For occasions when the mathematical model predicts oil flow at the pistons exceeding oil flow in the journal bearing, the pistons will not move as predicted. Instead, the inertia force acting on the pistons will tend to move them in a direction away from the balance position until an oil supply large enough to feed the pistons is again available. Also wear problems could exist because of the lack of oil to prevent metal-to-metal contact.

The bearings in a machine subjected to frequent start-stop operation, as in a washing machine, are much more vulnerable to wear than those in a machine constantly running at high speed. This is so because at low speed, there isn't enough fluid pumped between the shaft and bearing to generate pressures large enough to support the gravitational load and prevent metal-to-metal contact. The effect of this wear is to increase the clearance altering the response of the balancing system, and thus the designer should consider a wear-preventive coating on the bearing surface. The advantage of the three-support design is that wear in the journal bearing can be greatly reduced by placing it concentrically around the shaft when the machine is at rest.

The adaptation of this balancing system to the Whirlpool horizontal axis combination washer/dryer requires special design considerations for machine operation during the wash cycle during which clothes are tumbled at about 40 rpm. The amount of unbalance in a previous wash determines the location of the tub relative to the shaft at start. Either a special mechanism must be included to center the tub at the beginning of every run or else the design of the balance system must include considerations for balance at such a low speed. This balancing system is also applicable to vertical axis machines with considerations for rendering the pistons motionless during the agitate cycle of each wash. Wear due to the downward-acting force of gravity is eliminated.

In the two-bearing design success was found placing the bearings about four inches apart. The ball bearings in the current machine are about 5.6 inches apart. This, along with the fact that increased tub weight favorably helps the balancing system, leads to the possibility of an enlarged tub in the same size machine frame, thereby increasing capacity. Consideration must be given, however, to the additional space needed in the frame for the tub to rotate eccentrically about the shaft. This system also requires a pump to supply oil to the bearing and an oil sump to collect oil lost to leakage. An investigation of the temperature rise in the bearing and the ability of the oil to carry away heat is necessary. These considerations and others are all a necessary part of the final design of the proposed balancing mechanism.

The use of a journal bearing as the basis of a mechanism to sense the direction and amount of unbalance has been shown to be feasible. The selection of design parameters is governed by the range of loads and speeds encountered by the particular machine. The modification of this system to apply to a particular balancing problem is left to the ingenuity of the designer.

Recommendations

The solutions presented here were obtained by numerical analysis and therefore contain an inherent degree of inaccuracy. Further study is necessary to correlate the solutions predicted mathematically here with data obtained from a prototype of the design. The design of such a model should be based on the ability to vary as many of the parameters discussed earlier as possible. The radius of the shaft, surface area of the pistons, and clearance between the bearings and shaft are among the more difficult to experimentally vary. The selection of these should be based on results obtained from this analysis. The other primary design parameters--fluid viscosity, machine acceleration, distance between support bearings, losses in hydraulic lines, and total tub weight can be varied with less difficulty. This analysis should again be the principal tool on which the selection and variation of these parameters is based.

APPENDIX

APPENDIX

Introduction

The computer program is presented here as a tool for any future work. As written the program includes the following fixed variables:

Time Interval
Between Iterations = .05 sec

Number of Iterations = 800

Maximum Spin Speed = 600 rpm

The remainder of the design parameters are read in the main program and printed on the first page of output as shown in the first page following this introduction. The computer variable name for each design parameter is enclosed in parentheses to the right of each statement in this print-out. Following this is the actual computer program written in FORTRAN IV programming language and consisting of one main program and eight subprograms.

```
THE VARIABLE NAME IS GIVEN BELOW IN PARINTHESES

WASHER DESIGN      IS IN OPERATION (DESIGN)
LOCATION OF CLOTHES FROM CYLINDER 1 IS     DEGREES (THETB)
THE NUMBER OF ITERATIONS IS     (LIMIT)
THE TIME INTERVAL IS     SECONDS, (HT)
THE VISCOSITY OF THE FLUID IS       REYNS  (VISCOS)
THE CLEARANCE BETWEEN BEARING AND SHAFT IS     INCHES (C)
TUB MOTION NEEDED FOR BALANCE IS     INCHES (TUBMO)
THE WEIGHT DENSITY OF THE FLUID IS     LBF/CUBIC INCHES (GAMMA)
THE DIAMETER OF LINE 1 IS     INCHES(D1)
THE LENGTH OF LINE 1 IS     INCHES (L1)
THE LENGTH OF THE CAPILLARY IS     INCHES(LC)
THE DIAMETER OF THE CAPILLARY  IS     INCHES(DC)
THE LENGTH OF LINE 2 IS     INCHES (L2)
THE DIAMETER OF LINE 2 IS     INCHES(D2)
THE INNER RADIUS OF THE ELASTIC TUBE IS     INCHES (RI)
THE OUTER RADIUS OF THE ELASTIC TUBE IS     INCHES (RO)
THE LENGTH OF THE ELASTIC TUBE IS     INCHES (LB)
POISSONS RATIO FOR THE ELASTIC TUBE IS     (XLAMBA)
THE ELASTIC MODULUS OF THE ELASTIC TUBE IS     PSI (E)
THE THICKNESS OF THE ELASTIC TUBE IS     INCHES (T)
THE ACCELERATION OF THE MACHINE IS     REV/MIN**2 (ACC)
DISTANCE FROM BALL BEARING TO JOURNAL BEARING IS     INCHES (BS)
DISTANCE FROM BALL BEARING TO CENTER OF TUB IS     INCHES (Z)
THE WEIGHT OF UNBALANCED CLOTHES IS     LBF (WACLO)
THE WEIGHT OF TUB, BALANCED CLOTHES, AND WATER IS     LBF (WATUB)
THE RADIUS OF UNBALANCED CLOTHES IS     INCHES (WASITE)
THE LENGTH OF THE SENSOR BEARING IS     INCHES (ALEN)
THE DIAMETER OF THE SENSOR BEARING IS     INCHES (D)
THE RADIUS OF THE SHAFT IS     INCHES (R)
THE SURFACE AREA OF THE PISTONS IS     IN**2 (AREA)
MOTION OF PISTON 1 NEEDED FOR BALANCE IS     INCHES (TUBM1)
MOTION OF PISTON 2 NEEDED FOR BALANCE IS     INCHES (TUBM2)
THE LOSS PARAMETER IS     SEC (XLOSS)
```

```
-RUN FORTN,51E25619,TIGNER-J,1,100
-PWRD MEB6KQ
-FOR,IS POOL
      PARAMETER LIMIT=800
      REAL L1,L2,LB,LC
      REAL K11,K12,K13,K14,K21,K22,K23,K24,K31,K32,K33,K34,K41,K42,K43,
     CK44
      INTEGER DESIGN
      DIMENSION X1(LIMIT),Y1(LIMIT),X2(LIMIT),Y2(LIMIT),Q(LIMIT)
     C,PRMAX(LIMIT)
      COMMON /BL1/WACLO,WASITE,WATUB,VISCOS,R,  ALEN,D,RAT(10),RATZ(10)
     C,ACC,B,A,Z,BS,ROLL
      COMMON /BL3/PSI(12)
      COMMON /BL4/PRES(10)
      COMMON /BL5/PSTAR(11),PF(11,21)
      COMMON /BL7/THETB,A1,GAMMA,LB,C1,C6,RB,C2,AREA,U,C7,    W1,W,C4,C
     C,THETA,TUBMO,THETC
      COMMON /BL8/DY1T,DY2T,SAM
      COMMON /BL9/SOMMER
      COMMON /BL11/EPP(10)
      COMMON /BL12/DESIGN
      COMMON /BL13/P1LAST(LIMIT),P2LAST(LIMIT),P3LAST(LIMIT),P4LAST(LIMI
     CT),
     CP1,P2,P3,P4,N,PPH,PPL,PPHLAS,PPLLAS
      COMMON /BL14/CY(5),CZ(5)
      DATA (PSTAR(IG),IG=1,11)/15.,20.,27.,35.,40.,44.,51.,60.,65.,180.,
     C12.0/
      DATA (PRES(IB),IB=1,10)/1.82,1.85,1.89,1.93,2.07,2.23,2.40,2.70,3
     C.17,4.07/
      DATA (PSI(ID),ID=1,12)/85.,79.5,74.0,68.5,63.1,56.9,50.6,44.0,36.2
     C,26.5,15.5,0./
      DATA (CY(IT),IT=1,5)/3.3,3.26,3.24,3.22,3.2/
      DATA (CZ(IL),IL=1,5)/4.1,3.8,3.6,3.5,3.4/
      DATA (RAT(IA),IA=1,9)/1.35,.632,.382,.261,.179,.120,.0765,.0448,.0
     C191/
      DATA (RATZ(IE),IE=1,10)/19.6,6.86,4.28,3.54,2.51,2.21,1.84,1.69,1.
     C56,1.35/
      DATA (EPP(IS),IS=1,10)/.64,.46,.35,.31,.26,.22,.20,.18,.16,.15/
      READ(5,306)DESIGN,C,THETB,ROLL
  306 FORMAT(I5,3F10.5)
      READ(5,303)HT,GAMMA,D1,L1,LC,DC,L2
      READ(5,304)D2,RO,RI,XLAMBA,T,LB
      READ(5,305)E,ACC,BS
      READ(5,20)A,B,Z,WACLO,WATUB,WASITE,ALEN
      READ(5,21)D,R,VISCOS,AREA
      H=HT
      DO 777 L=1,21
      READ(5,776)(PF(K,L),K=1,11)
      TUBMO=WACLO*WASITE/(WATUB+WACLO)
  777 CONTINUE
      WRITE(6,819)
      U=AREA
      A1=3.14159*(D1**2)/4.
      C1=128.*L1*VISCOS/(GAMMA*D1**2)
      C2=128.*VISCOS*LC*U/(GAMMA*3.14159*DC**4)
      C4=128.*L2*VISCOS/(GAMMA*D2**2)
      C6=RI*(1.-XLAMBA**2)*RI/(E*T)
      C7=4.*3.14159**2/(386.*3600.)
```

```
      XLOSS=GAMMA*(C2+(C1+C4)*U/A1)

      INITIAL CONDITIONS AND PROMINENT VARIABLE SUBSTITUTIONS
      JTEST=1
      JQ=0.
      TSTART=.95
      TIME=TSTART
      RB=RI
      W1=WACLO
      W2=WATUB-WACLO
      W=WASITE
      THETA=THETB
      THETC=THETB+180.
      THETBR=THETB*3.14159/180.
      TUBM1=TUBMO*COS(THETBR)
      TUBM2=TUBMO*SIN(THETBR)
      X1(1)=0.
      X2(1)=0.
      Y1(1)=0.
      Y2(1)=0.
      WRITE(6,848)DESIGN
      WRITE(6,851)THETB
      LIM=LIMIT
      WRITE(6,850)LIM
      WRITE(6,820)HT
      WRITE(6,846)VISCOS
      WRITE(6,849)C
      WRITE(6,847)TUBMO
      WRITE(6,821)GAMMA
      WRITE(6,822)D1
      WRITE(6,823)L1
      WRITE(6,824)LC
      WRITE(6,825)DC
      WRITE(6,826)L2
      WRITE(6,827)D2
      WRITE(6,828)RI
      WRITE(6,829)RO
      WRITE(6,830)LB
      WRITE(6,831)XLAMBA
      WRITE(6,832)E
      WRITE(6,833)T
      WRITE(6,834)ACC
      IF(DESIGN .EQ. 2)GO TO 80
      WRITE(6,835)A
      WRITE(6,836)B
      GO TO 81
   80 WRITE(6,852)BS
   81 CONTINUE
      WRITE(6,837)Z
      WRITE(6,838)WACLO
      WRITE(6,839)WATUB
      WRITE(6,840)WASITE
      WRITE(6,841)ALEN
      WRITE(6,842)D
      WRITE(6,843)R
      WRITE(6,844)AREA
      WRITE(6,855)TUBM1
      WRITE(6,856)TUBM2
      WRITE(6,853)XLOSS
      WRITE(6,845)C1,C2,C4,C6,C7
```

```
      RUNGE-KUTTA METHOD OF SOLVING TWO 2ND-ORDER DIFF  EQS
      DO 10 N=2,LIMIT
      HH=H
      K11=Y1(N-1)
      K31=Y2(N-1)
      CALL TUB(TIME,X1(N-1),Y1(N-1),X2(N-1),Y2(N-1),HH)
      P1LAST(N-1)=P1
      P2LAST(N-1)=P2
      P3LAST(N-1)=P3
      P4LAST(N-1)=P4
      Q(N-1)=FLOW(SOMMER)
      PRMAX(N-1)=SAM
      K21=DY1T
      K41=DY2T
      K12=Y1(N-1)+H*K21/2.
      K32=Y2(N-1)+H*K41/2.
      A9=TIME+H/2.
      A2=X1(N-1)+H*K11/2.
      A3=Y1(N-1)+H*K21/2.
      B1=X2(N-1)+H*K31/2.
      B2=Y2(N-1)+H*K41/2.
      HH=H/2.
      CALL TUB(A9,A2,A3,B1,B2,HH)
      K22=DY1T
      K42=DY2T
      K13=Y1(N-1)+H*K22/2.
      K33=Y2(N-1)+H*K42/2.
      A4=X1(N-1)+H*K12/2.
      A5=Y1(N-1)+H*K22/2.
      B3=X2(N-1)+H*K32/2.
      B4=Y2(N-1)+H*K42/2.
      CALL TUB(A9,A4,A5,B3,B4,HH)
      K23=DY1T
      K43=DY2T
      K14=Y1(N-1)+H*K23
      K34=Y2(N-1)+H*K43
      A6=TIME+H
      A7=X1(N-1)+H*K13
      A8=Y1(N-1)+H*K23
      B5=X2(N-1)+H*K33
      B6=Y2(N-1)+H*K43
      HH=H
      CALL TUB(A6,A7,A8,B5,B6,HH)
      K24=DY1T
      K44=DY2T
      X1(N)=X1(N-1)+H*(K11+2.*K12+2.*K13+K14)/6.
      Y1(N)=Y1(N-1)+H*(K21+2.*K22+2.*K23+K24)/6.
      X2(N)=X2(N-1)+H*(K31+2.*K32+2.*K33+K34)/6.
      Y2(N)=Y2(N-1)+H*(K41+2.*K42+2.*K43+K44)/6.
      TIME=TIME+H
   10 CONTINUE
      WRITE(6,65)
      Q(LIMIT)=Q(LIMIT-1)*(ACC*TIME/60.)/ (ACC*(TIME-H)/60.)
   65 FORMAT(/  ,5X,- TIME         SPEED        FINERT        X1            X
     C2            Y1            Y2          Q NEEDED  Q AVAILABLE-,/)
      TIME=TSTART+HT
      H=HT
      DO 60 N=2,LIMIT,2
      S=ACC*TIME/60.
```

```
      IF(S .GT. 600.)S=600.
      S1=WASITE*SIN(THETBR)-X2(N)
      S2=WASITE*COS(THETBR)-X1(N)
      FIX1=C7*S**2*(WACLO*S2-WATUB*X1(N))
      FIX2=C7*S**2*(WACLO*S1-WATUB*X2(N))
      FINERT=SQRT(FIX1**2+FIX2**2)
      Q1=ABS(U*Y1(N))
      Q2=ABS(U*Y2(N))
      QA=Q(N)*R*C*S*ALEN*3.14159/30.
      QN=Q1+Q2
      IF(QN .GT. QA)GO TO 900
      WRITE(6,61)TIME,S     ,FINERT,X1(N),X2(N),Y1(N),Y2(N),QN,QA
     C,PRMAX(N)
61    FORMAT(1X,10E12.5)
      GO TO 902
900   WRITE(6,901)TIME,S,FINERT,X1(N),X2(N),Y1(N),Y2(N),QN,QA
     C,PRMAX(N)
901   FORMAT(1X,10E12.5,-  *  -)
902   CONTINUE
      TIME=TIME+2.*H
60    CONTINUE
      WRITE(6,65)
50    CONTINUE
 20   FORMAT(7F10.5)
 21   FORMAT(2F10.5,F10.7,F10.5)
 31   FORMAT(1X,-VISCOSITY = -,F10.7,- REYNS                  TUB MOTION
     CNEEDED FOR BALANCE = -,F5.3,- INCHES-,//)
303   FORMAT(7F10.4)
304   FORMAT(6F10.4)
305   FORMAT(3F10.6)
776   FORMAT(11F5.4)
819   FORMAT(-1-,1X,-THE VARIABLE COMPUTER NAME IS GIVEN BELOW IN PARENT
     CHESES-,/)
820   FORMAT(1X,-THE TIME INTERVAL IS -,F5.2,- SECONDS, (HT)-)
821   FORMAT(1X,-THE WEIGHT  DENSITY  OF THE FLUID IS -,F7.4,- (GAMMA)
     C LBF/CUBIC INCHES-)
822   FORMAT(1X,-THE DIAMETER OF LINE 1 IS -,F7.3,- INCHES (D1)-)
823   FORMAT(1X,-THE LENGTH OF LINE 1 IS -,F7.3,- INCHES (L1)-)
824   FORMAT(1X,-THE LENGTH OF THE CAPILLARY IS -,F7.3,- INCHES (LC)-)
825   FORMAT(1X,-THE DIAMETER OF THE CAPILLARY IS -,F7.4,- INCHES (DC)-)
826   FORMAT(1X,-THE LENGTH OF LINE 2 IS -,F7.3,- INCHES (L2)-)
827   FORMAT(1X,-THE DIAMETER OF LINE 2 IS-, F7.3,- INCHES (D2)-)
828   FORMAT(1X,-THE INNER RADIUS OF THE ELASTIC TUBE IS -,F7.4,- INCHES
     C (RI)-)
829   FORMAT(1X,-THE OUTER RADIUS OF THE ELASTIC TUBE IS -,F7.4,- INCHES
     C (RO)-)
830   FORMAT(1X,-THE LENGTH OF THE ELASTIC TUBE IS -,F7.3,- INCHES (LB)-
     C)
831   FORMAT(1X,-POISSON,S RATIO FOR THE ELASTIC TUBE IS -,F10.6,- (XLAM
     CBA)-)
832   FORMAT(1X,-THE ELASTIC MODULUS OF THE ELASTIC TUBE IS -,F12.2,- PS
     CI (E)-)
833   FORMAT(1X,-THE THICKNESS OF THE ELASTIC TUBE IS- ,F7.4,-(T)-)
834   FORMAT(1X,-THE ACCELERATION OF THE MACHINE IS -,F9.2,- REV/MIN**2
     C(ACC)-)
835   FORMAT(1X,-DISTANCE FROM BALL BEARING TO SENSOR BEARING IS -,F5.2,
     C,- INCHES (A)-)
836   FORMAT(1X,-DISTANCE BETWEEN BALL BEARINGS IS -,F5.2,- INCHES (B)-)
837   FORMAT(1X,-DISTANCE FROM BALL BEARING TO CENTER OF TUB IS -,F5.2,
     C- INCHES (Z)-)
```

```
838 FORMAT(1X,-THE WEIGHT OF UNBALANCED CLOTHES IS -,F5.2,-
   C LBF (WACLO)-)
839 FORMAT(1X,-THE WEIGHT OF TUB,BALANCED CLOTHES, AND WATER IS -,F5.2
   C,- LBF (WATUB)-)
840 FORMAT(1X,-THE RADIUS OF UNBALANCED CLOTHES IS -,F5.2,- INCHES (WA
   CSITE)-)
841 FORMAT(1X,-THE LENGTH OF THE SENSOR BEARING IS -,F6.4,- INCHES (AL
   CEN)-)
842 FORMAT(1X,-THE DIAMETER OF THE SENSOR BEARING IS -,F6.4,- INCHES (
   CD)-)
843 FORMAT(1X,-THE RADIUS OF THE SHAFT IS -,F8.5,- INCHES (R)-)
844 FORMAT(1X,-THE SURFACE AREA OF THE PISTONS IS -,F8.5,- IN**2 (AREA
   C)-)
845 FORMAT(/,1X,5E12.5,- THESE ARE LOSS CONSTANTS-,/)
846 FORMAT(1X,-THE VISCOSITY OF THE FLUID IS -,F10.7,- REYNS (VISCOS
   C)-)
847 FORMAT(1X,-TUB MOTION NEED FOR BALANCE IS -,F5.3,- INCHES (TUBMO)-
   C)
848 FORMAT (1X,    -WASHER DESIGN -,I2,- IS IN OPERATION (DESIGN)-)
849 FORMAT(1X,-THE CLEARANCE BETWEEN BEARING AND SHAFT IS -,F7.4,
   C- INCHES (C)-)
850 FORMAT(1X,-THE NUMBER OF ITERATIONS IS -,I4  ,- (LIMIT)-)
851 FORMAT(1X,-LOCATION OF CLOTHES FROM CYLINDER 1 IS -,F4.0,- DEGREES
   C (THETB) -)
852 FORMAT(1X,-DISTANCE FROM BALL BEARING TO JOURNAL BEARING IS -
   C,F5.2,- INCHES (BS)-)
853 FORMAT(1X,-THE LOSS PARAMETER IS -,F10.5,- SEC (XLOSS)-)
854 FORMAT(1X,-THE ANGLE OF ROLL IS -,F5.1,- DEGREES (ROLL)-)
855 FORMAT(1X,-MOTION OF PISTON 1 NEEDED FOR BALANCE IS -,F5.2,- INCH
   CES (TUBM1)-)
856 FORMAT(1X,-MOTION OF PISTON 2 NEEDED FOR BALANCE IS -,F5.2,- INCH
   CES (TUBM2)-)
    END


    SUBROUTINE 1

-FOR,IS TUB
    SUBROUTINE TUB(TIME,X1,Y1,X2,Y2,HH)
    PARAMETER LIMIT=800
    REAL LB
    INTEGER DESIGN
    COMMON /BL12/DESIGN
    COMMON /BL1/WACLO,WASITE,WATUB,VISCOS,R,  ALEN,D,RAT(10),RATZ(10)
   C,ACC,B,A,Z,BS,ROLL
    COMMON /BL7/THETB,A1,GAMMA,LB,C1,C6,RB,C2,AREA,U,C7,    W1,W,C4,C
   C,THETA,TUBMO,THETC
    COMMON /BL8/DY1T,DY2T,SAM
    COMMON /BL13/P1LAST(LIMIT),P2LAST(LIMIT),P3LAST(LIMIT),P4LAST(LIMI
   CT),
   CP1,P2,P3,P4,N,PPH,PPL,PPHLAS,PPLLAS
    XMOM=3.1416*D**4/64.
    THETBR=THETB*3.14159/180.
    S=ACC*TIME/60.
    IF(S .GT. 600.)S=600.
    S1=WASITE*SIN(THETBR)-X2
    S2=WASITE*COS(THETBR)-X1
    FIX1=C7*S**2*(WACLO*S2-WATUB*X1)
    FIX2=C7*S**2*(WACLO*S1-WATUB*X2)
```

```
      FINERT=SQRT(FIX1**2+FIX2**2)
      FF=ABS(FIX1)
      IF(FF .LE. 0.)GO TO 60
      ANGRAD=ATAN(FIX2/FIX1)
      IF(FIX1 .LT. 0.)GO TO 61
      THETAD=180.*ANGRAD/3.14159
      GO TO 62
 61   THETAD=180.+180.*ANGRAD/3.14159
      GO TO 62
 60   THETAD=270.
      IF(FIX1 .GT. 0.)THETAD=90.
 62   CONTINUE
      IF(THETAD .LT. 0.)THETAD=THETAD+360.
      IF(DESIGN .EQ. 2)GO TO 10
      DEL1=FINERT*(Z-B)*A*B*(1.-A**2/B**2)/(6.*30.*10**6*XMOM)
      R2=DEL1*6.*30.*10**6*XMOM*B/((B-A)*A*(2.*B*A -2.*A**2))
      XLOAD=R2/(ALEN*D)
      BOB=ALTER(XLOAD,C,TIME)
      GO TO 11
 10   FORCE=FINERT*Z/BS
      XLOAD=FORCE/(ALEN*D)
      BOB=ALTER(XLOAD,C,TIME)
 11   CONTINUE
      TOM=ALOAD(TIME,C)
      BILL=ATT(BOB)
      SAM=PMAX(BOB,TOM)
      XNA1=180.-(BILL+THETAD+ROLL)
      IF(XNA1 .LT. 0.)XNA1=XNA1+360.
      P1=PRESS(BOB,SAM,XNA1)
      XNA2=270.-(BILL+THETAD+ROLL)
      IF(XNA2 .LT. 0.)XNA2=XNA2+360
      P2=PRESS(BOB,SAM,XNA2)
      XNA3=360.-(BILL+THETAD+ROLL)
      IF(XNA3 .LT. 0.)XNA3=XNA3+360.
      P3=PRESS(BOB,SAM,XNA3)
      XNA4=90. -(BILL+THETAD+ROLL)
      IF(XNA4 .LT . 0.)XNA4=XNA4+360.
      P4=PRESS(BOB,SAM,XNA4)
      PPH=P1
      PPL=P3
      PPHLAS=P1LAST(N-1)
      PPLLAS=P3LAST(N-1)
      DY1T=TEST(TIME,X1,Y1,FIX1,HH)
      PPH=P2
      PPL=P4
      PPHLAS=P2LAST(N-1)
      PPLLAS=P4LAST(N-1)
      DY2T=TEST(TIME,X2,Y2,FIX2,HH)
      RETURN
      END


      SUBROUTINE 2

-FOR,IS TEST
      FUNCTION TEST(TIME,X,Y,FINERT,HH)
      PARAMETER LIMIT=800
      REAL LB
      REAL MT
```

```
      COMMON /BL1/WACLO,WASITE,WATUB,VISCOS,R,  ALEN,D,RAT(10),RATZ(10)
     C,ACC,B,A,Z,BS
      COMMON /BL7/THETB,A1,GAMMA,LB,C1,C6,RB,C2,AREA,U,C7,    W1,W,C4,C
     C,THETA,TUBMO,THETC
      COMMON /BL13/P1LAST(LIMIT),P2LAST(LIMIT),P3LAST(LIMIT),P4LAST(LIMI
     CT),
     CP1,P2,P3,P4,N,PPH,PPL,PPHLAS,PPLLAS
      TUBMO=WACLO*WASITE/(WATUB+WACLO)
      YY=Y
      XX=X
      G=AREA
      PH=PPH
      PL=PPL
      PHLAST=PPHLAS
      PLLAST=PPLLAS
      PH1P=(PH-PHLAST)/HH
      PL1P=(PL-PLLAST)/HH
      MT    =WATUB/32.2
      IF(YY .LT. 0.)GO TO 10
      GO TO 11
   10 PP=PH
      PH=PL
      PL=PP
      PH1P=(PH-PLLAST)/HH
      PL1P=(PL-PHLAST)/HH
      YY=-YY
      G=-G
   11 CONTINUE
      PINA       =(A1*PH        /GAMMA-2.*3.14159*LB*C1*C6*RB*PH1P
     C+   (YY) *(-A1*C2-2.*3.14159*LB*GAMMA*C6**2*PH1P        *C1*(C2+
     CC4*U/A1)-(C1+C4)*U))/(A1/GAMMA+2.*3.14159*LB*C1*C6**2*PH1P        )
      POTA       =(A1*PL        /GAMMA-2.*3.14159*LB*RB*C1*C6*PL1P
     C+   (YY) *(A1*C2+2.*3.14159*LB*+1*C6**2*GAMMA*PL1P        *(C2
     C+C4*U/A1)+(C1+C4)*U))/(A1/GAMMA+2.*3.1415 *LB*C1*C6**2*PL1P
     C)
      IF(PINA .LT. 0.)PINA=0.
      IF(POTA .LT. 0.)POTA=0.
  714 CONTINUE
      XLOSS=YY      *GAMMA*(C2+(C4+C1)*U/A1)
      DIFF=PH-PL
  717 CONTINUE
      DYDT       = (G/MT    )*(PINA       -POTA       )-FINERT/MT
  582 CONTINUE
      TEST=DYDT
      RETURN
      END


      SUBROUTINE 3

-FOR,IS ALTER
      SUBROUTINE ALTER(XLOAD ,C,TIME)
      COMMON /BL1/WACLO,WASITE,WATUB,VISCOS,R,  ALEN,D,RAT(10),RATZ(10)
     C,ACC,B,A,Z,BS
      COMMON /BL9/SOMMER
      COMMON /BL11/EPP(10)
      SSTAR=100.
      S=ACC*TIME/60.
      IF(S .GT. 600.)S=600.
```

```
      SOMMER=(VISCOS*S*R**2*2.*3.14159)/(XLOAD*60.*C**2)
   60 CONTINUE
      IF(SOMMER .GT. SSTAR)GO TO 50
      GO TO 61
   50 SSTAR=SSTAR+100.
      GO TO 60
   61 CONTINUE
      IF(SOMMER .LT. 1.)GO TO 80
      IF(SOMMER .GT. 19.6)GO TO 2
      IF(SOMMER .GT. 6.86)GO TO 3
      IF(SOMMER .GT. 4.28)GO TO 4
      IF(SOMMER .GT. 3.54)GO TO 5
      IF(SOMMER .GT. 2.51)GO TO 6
      IF(SOMMER .GT. 2.21)GO TO 7
      IF(SOMMER .GT. 1.84)GO TO 8
      IF(SOMMER .GT. 1.69)GO TO 9
      IF(SOMMER .GT. 1.56)GO TO 10
      IF(SOMMER .GT. 1.35)GO TO 11
      IF(SOMMER .GT. 1.00)GO TO 12
      GO TO 99
   80 IF(SOMMER .LT. .1)GO TO 81
      IS=10.*SOMMER+1.-.99999
      SS=IS/10.
      ECCRAT=EPP(IS+1)-(EPP(IS+1)-EPP(IS))*ALOG10((SS+.1)/SOMMER)/ALOG
     C10((SS+.1)/SS)
      GO TO 99
   81 ECCRAT=.64-(.64-1.)*(.1-SOMMER)/(.1-0.)
      GO TO 99
    2 ECCRAT=0.-(0.-.01)*ALOG10(SSTAR/SOMMER)/ALOG10(SSTAR/19.6)
      GO TO 99
    3 ECCRAT=.01-(.01-.02)*ALOG10(19.6/SOMMER)/ALOG10(19.6/6.86)
      GO TO 99
    4 ECCRAT=.02-(.02-.03)*ALOG10(6.86/SOMMER)/ALOG10(6.86/4.28)
      GO TO 99
    5 ECCRAT=.03-(.03-.04)*ALOG10(4.28/SOMMER)/ALOG10(4.28/3.54)
      GO TO 99
    6 ECCRAT=.04-(.04-.05)*ALOG10(3.54/SOMMER)/ALOG10(3.54/2.51)
      GO TO 99
    7 ECCRAT=.05-(.05-.06)*ALOG10(2.51/SOMMER)/ALOG10(2.51/2.21)
      GO TO 99
    8 ECCRAT=.06-(.06-.07)*ALOG10(2.21/SOMMER)/ALOG10(2.21/1.84)
      GO TO 99
    9 ECCRAT=.07-(.07-.08)*ALOG10(1.84/SOMMER)/ALOG10(1.84/1.69)
      GO TO 99
   10 ECCRAT=.08-(.08-.09)*ALOG10(1.69/SOMMER)/ALOG10(1.69/1.56)
      GO TO 99
   11 ECCRAT=.09-(.09-.1)*ALOG10(1.56/SOMMER)/ALOG10(1.56/1.33)
      GO TO 99
   12 ECCRAT=.1-(.1-.15)*ALOG10(1.33/SOMMER)/ALOG10(1.33/1.0)
   99 CONTINUE
      ALTER=ECCRAT
      RETURN
      END
```

SUBROUTINE 4

```
-FOR,IS ALOAD
      FUNCTION ALOAD(TIME,C)
```

```
      COMMON /BL1/WACLO,WASITE,WATUB,VISCOS,R,  ALEN,D,RAT(10),RATZ(10)
     C,ACC,B,A,Z,BS
      COMMON /BL9/SOMMER
      S=ACC*TIME/60.
      IF(S .GT. 600.)S=600.
      XLOAD=(VISCOS*S*R**2*2.*3.14159)/(SOMMER*60.*C**2)
      ALOAD=XLOAD
      RETURN
      END
```

SUBROUTINE 5

```
-FOR,IS FLOW
      FUNCTION FLOW(SOMMER)
      COMMON /BL14/CY(5),CZ(5)
      IF(SOMMER .GT. 100.)GO TO 1
      IF(SOMMER .GT. 10.)GO TO 2
      IF(SOMMER .LT. 2.)GO TO 3
      IT=SOMMER/2.+.00001
      ZT=2.*IT
      FRAT=CY(IT+1)-(CY(IT+1)-CY(IT))*ALOG10((ZT+2.)/SOMMER)/ALOG10((Z(
     C+2.)/ZT)
      GO TO 4
    1 FRAT=3.1
      GO TO 4
    2 FRAT=3.1-(3.1-3.2)*ALOG10(100./SOMMER)/ALOG10(100./10.)
      GO TO 4
    3 IF(SOMMER .LT. 1.)GO TO 5
      FRAT=3.3-(3.3-3.4)*ALOG10(2./SOMMER)/ALOG10(2./1.)
      GO TO 4
    5 IF(SOMMER .LT. .2)GO TO 7
      IL=10.*SOMMER/2.+.00001
      ZL=2.*IL/10.
      FRAT=CZ(IL+1)-(CZ(IL+1)-CZ(IL))*ALOG10((ZL+.2)/SOMMER)/ALOG10((
     CZL+.2)/ZL)
      GO TO 4
    7 IF(SOMMER .LT. .1)GO TO 8
      FRAT=4.1-(4.1-4.4)*ALOG10(.2/SOMMER)/ALOG10(.2/.1)
      GO TO 4
    8 FRAT=4.5
    4 CONTINUE
      FLOW=FRAT
      RETURN
      END
```

SUBROUTINE 6

```
-FOR,IS ATT
      FUNCTION ATT(DUMD)
      COMMON /BL3/PSI(12)
      ID=    DUMD*10.+1.-.99999
      ZD=ID/10.
      PSIX=PSI(ID+2)-((ZD+.1)-    DUMD)*(PSI(ID+2)-PSI(ID+1))/((ZD+.1)-ZD
     C)
      ATT=PSIX
      RETURN
      END
```

SUBROUTINE 7

```
-FOR,IS PMAX
      FUNCTION PMAX(DUME,DUMF)
      COMMON /BL4/PRES(10)
      ERATIO=DUME
      XLOAD=DUMF
C     COMPUTE THE MAXIMUM PRESSURE

      IF(ERATIO .LT. .8)GO TO 37
      IF (ERATIO .LT. .9) GO TO 79
      IF (ERATIO .LT. .95) GO TO 80
      IF (ERATIO .LT. .97) GO TO 81
      IF(ERATIO .LT. 1.)GO TO 82
   37 IB=10.*ERATIO-.99999+1.
      ZB=IB/10.
      PMAXR=10.**(ALOG10(PRES(IB+2))-((ZB+.1)-ERATIO)*ALOG10(PRES(IB+2)/
     CPRES(IB+1))/((ZB+.1)-ZB))
      GO TO 98
   79 PMAXR=10.**(ALOG10(4.07)-(.9-ERATIO)*ALOG10(4.07/3.17)/(.9-.8))
      GO TO 98
   80 PMAXR=10.**(ALOG10(5.32)-(.95-ERATIO)*ALOG10(5.32/4.07)/(.95-.9))
      GO TO 98
   81 PMAXR=10.**(ALOG10(6.58)-(.97-ERATIO)*ALOG10(6.58/5.32)/(.97-.95))
      GO TO 98
   82 PMAXR=10.**(1.-(1.-ERATIO)*ALOG10(10./6.58)/(1.-.97))
   98 CONTINUE
      PMAX=XLOAD*PMAXR
      RETURN
      END
```

SUBROUTINE 8

```
-FOR,IS PRESS
      FUNCTION PRESS(DUMG,DUMH,DUMI)
      COMMON /BL5/PSTAR(11),PF(11,21)
      EPS=DUMG
      PMAX=DUMH
      WL=DUMI
      IF(WL .LT. 0.)WL=360.+WL
      IF(WL .GT. 210.)GO TO 660
      GO TO 661
  660 PTIALT=0.
      GO TO 198
  661 THETA=WL
      IF(EPS .LT. .1)GO TO 234
      IG=EPS*10.-.99999+1.
      ZG=IG/10.
      PSSAR=10.**(ALOG10(PSTAR(IG+1))-ALOG10((ZG+.1)/EPS)*ALOG10(PSTAR(I
     CG+1)/PSTAR(IG))/ALOG10((ZG+.1)/ZG))
      IF(THETA .LE. PSSAR)GO TO 660
      IH=THETA/10.-.99999+1.
      ZH=10.*IH
      IF(PF(IG,IH) .LE. 0.)GO TO 10
      PZ=10.**(ALOG10(PF(IG,IH+1))-ALOG10((ZH+10.)/THETA)*ALOG10(PF(IG,I
     CH+1)/PF(IG,IH))/ALOG10((ZH+10.)/ZH))
      GO TO 11
   10 PZ=PF(IG,IH+1)-((ZH+10.)-THETA)*(PF(IG,IH+1)-PF(IG,IH))/((ZH+10.)
     C-ZH)
```

```
 11  CONTINUE
     IF(PF(IG+1,IH) .LE. 0.)GO TO 12
     PQ=10.**(ALOG10(PF(IG+1,IH+1))-ALOG10((ZH+10.)/THETA)*ALOG10(PF(IG
    C+1,IH+1)/PF(IG+1,IH))/ALOG10((ZH+10.)/ZH))
     GO TO 13
 12  PQ=PF(IG+1,IH+1)-((ZH+10)-THETA)*(PF(IG+1,IH+1)-PF(IG+1,IH))/
    C((ZH+10.)-ZH)
 13  CONTINUE
 235 PTIALT=10.**(ALOG10(PQ)-ALOG10((ZG+.1)/EPS)*ALOG10(PQ/PZ)/ALOG10((
    CZG+.1)/ZG))
     GO TO 115
 234 IF(EPS .LT. .01)EPS=.01
     PSSAR=10.**(ALOG10(PSTAR(    1))-ALOG10((    .1)/EPS)*ALOG10(PSTAR(
    C  1)/PSTAR(11))/ALOG10(.1/.01))
     IF(THETA .LE. PSSAR)GO TO 660
     IH=THETA/10.-.99999+1.
     ZH=10.*IH
     PZ=10.**(ALOG10(PF(11,IH+1))-ALOG10((ZH+10.)/THETA)*ALOG10(PF(11,I
    CH+1)/PF(11,IH))/ALOG10((ZH+10.)/ZH))
     PQ=10.**(ALOG10(PF(    1,IH+1))-ALOG10((ZH+10.)/THETA)*ALOG10(PF( 1
    C  ,IH+1)/PF(    1,IH))/ALOG10((ZH+10.)/ZH))
     PTIALT=10.**(ALOG10(PQ)-ALOG10(.1/EPS)*ALOG10(PQ/PZ)/ALOG10(.1/.01
    C))
 115 CONTINUE
 198 PRESS=PMAX*PTIALT
     RETURN
     END
-XQT
    2       .0006          45.            0.
      .05        .0311        .125           20.          1.5        .1000         20.
     .125        .15300      .1375            .4        .0310           2.
  1000000.       1100.         3.9
      2.8          5.6        14.7           6.0         70.0         11.0       1.100
    1.100          .55     .000060            .2
.0001   0.    0.    0.    0.    0.    0.    0.    0..0001.0001
   .1.0001.0001   0.    0.    0.    0.    0.    0..0001    .3
  .26   .12   .03.0001    0.    0.    0.    0.    0..0001    .5
  .47   .28   .18  .075.0001.0001   0.    0.    0..0001   .68
   .6   .42   .31   .18    .1   .04.0001    0.    0..0001    .8
  .72   .56   .42   .29   .19    .1   .04.0001.0001.0001   .92
  .82   .67   .53   .38   .29   .16   .09   .03   .01.0001   .97
  .93   .78   .65   .52    .4   .25   .16   .07   .02.0001   1.0
  .98   .88   .76   .63   .52   .35   .23   .11   .04.0001   .98
  .99   .95   .87   .73   .62   .39    .3   .16   .06.0001   .97
  .99   .99   .96   .83   .73   .56    .4   .24   .09.0001   .94
  .97   1.0   1.0   .94   .85    .7   .53   .35   .15.0001   .91
  .92   .98   .99   .99   .95   .83   .68   .49   .24.0001   .85
  .82   .91   .96   .99   1.0   .95   .86   .67   .39.0001   .78
  .74   .80   .87   .93   .98   1.0   .98   .87   .59.0001   .69
   .6   .65   .72   .79   .84   .93   .98   1.0   .89.0001   .58
  .44   .49   .56   .61   .64   .71    .8    .9   .99.0001   .44
   .3   .31   .34   .38   .40   .43   .46   .53   .521.000   .28
  .14   .16   .16   .16   .16   .16   .16   .16   .16.0001   .14
  .06   .04   .04   .04   .04   .04   .04   .04   .01.0001   .06
.0001.0001.0001.0001.0001.0001.0001.0001.0001.0001   .01
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